Signal Space Theory and Applications to Communications

- Communication is recovery of original vectors in the signal spaces –

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Abstract

This paper gives the conclusive chapters in the Signal Space theory and its applications to communications described in two previous papers for JCSAT 2016 and 2017. In the first paper [1] a formulation of the signal space was given and its applications to interferences cancellation based on least-mean-square output (LMSO) method were analyzed. A problem of trivial zero output for excessive number of cancelling paths and other defects of the LMSO method were clarified based on the signal space analysis. An improved LMSE method was proposed and described in signal space concepts.

In the second paper [2] the structure of the signal space was established based on Tangent Square Summation (TSS) theorem. The TSS theorem can be restated as Inverse SIRs summation theorem. The TSS theorem is effective to expand the signal space theory to include the thermal noise.

In this paper a brief summary of the signal space theory is given but more emphasis is put in its applications. The improved LMSE method is based on regeneration of the wanted signal which is the very objective of communications. For digital communications the regeneration of the wanted signal replica with high fidelity can be made by demodulation. For analog modulations it is generally difficult as the wave-shape of the desired signal is not a pri.o.ri known at the receiver. An exception is frequency modulation (FM) which can regenerate the wanted signal with improved signal-to-noise ratio (SNR) at the receiver in good SNR conditions.

In this paper a hard limiting (HL) is analyzed as a means to regenerate the wanted signal at the receiver with improved signal-to-interferences ratio (SIR). The SIR improvement of HL method is based on "small signal suppression effect" universally observed in signal transmission systems [3].

Applications of the improved LMSE method to dual-polarization radio communication systems are analyzed based on the signal space theory. The method can be readily generalized to multiple-inputs-multiple-output (MIMO) systems with greater numbers of signals. The stability conditions of the control loops for MIMO systems are clarified.

Contents

1. Signals and Signal space

1.1 Correlation of signals

Inner Product or Correlation;

Suppose we have two signals S1(t) and S2(t). Then we can define the inner product of those signals;

$$(S1(t), S2(t)) = [-1/2T,+1/2T] \int S1(t) \cdot S2^*(t)\, dt / T$$

where S2(t)* means the complex conjugate of S2(t)

The above inner products are also called **correlation** of the signals S1(t) and S2(t).

T is the time duration for the integral, or correlation measurement.

Power of signals;

The self-correlation of a signal S(t) is physically the **power** of the signal;

$$(S(t), S(t)) = \| S \|^2$$

where ‖ S ‖ is called the **norm** of the signal S(t).

The power of the signal is **normalized** if the norm is calibrated to be ‖ S ‖ =1.

Schwarz inequality;

Suppose we have two signals X(t) and Y(t). Then the correlation of X(t), Y(t) meets the Schwarz inequality;

$$|(X, Y)| \text{ = or < } \| X \| \cdot \| Y \|$$

Angle Between Signals in Signal Space;

The correlation or inner product between two signals X and Y can be expressed as follows;

$$(X, Y) / (\| X \| \cdot \| Y \|) = \cos(\theta) \cdot e^{(j\phi)}$$

where θ is the **angle** between vectors X and Y in the Signal Space and φ is the **phase** of the complex value (X,Y).

The amplitude of the above formula;

$$\cos(\theta) = |(X, Y)| / (\| X \| . \| Y \|)$$

is also called the **likelihood** of signals X and Y.

For θ = 0, the signals are **identical**; X = Y, or **totally correlated**.

For θ= π/2, $\cos(\theta) = |(X, Y)| / (\| X \| . \| Y \|) = 0$, the signals X and Y are totally **uncorrelated** or mutually **orthogonal** in the Signal Space.

1.2 Originality and Orthogonality

Signals from separate sources are mutually **original**. The original signals are mutually **uncorrelated** because they are modulated by independent source signals and their carriers are mutually incoherent. Thermal noises are orthogonal to any other signals as they are random and incoherent in nature.

The original signals are mutually orthogonal, but the converse is not true. Namely orthogonal signals are not necessarily original. Suppose we have two signals X and Y;

$$X = a.S1 + b.S2$$

$$Y = c.S1 + d.S2$$

where S1 and S2 are original signals with normalized amplitude.

Then X and Y can be orthogonal;

$$(X,Y)= a.c^* + b.d^* = 0$$

if the coefficients {a,b,c,d} meet the above equation.

1.3 Signal Space

Suppose we have signals Sd, S1, S2, ,,, Sm from different sources. Then they form a signal space with each signal giving the bases of the space. Without loss of generality, we can normalize their amplitude to 1. $\| Si \| = 1$ for all i.

The signal space is a vector space spanned by the original signals {Si ; i = 1,2,3,,,,m}.

Any signal in the communication system is a combination of those signals originating from different sources.

Suppose

$$X = x1 \cdot S1 + x2 \cdot S2 + ,,, xm \cdot Sm$$

$$Y = y1 \cdot S1 + y2 \cdot S2 + ,,, ym \cdot Sm$$

Then

$$(X,Y) = x1 \cdot y1^* + x2 \cdot y2^* +,,,,, xm \cdot ym^*$$

Thus the signals X and Y can be expressed as vectors in the signal space;

$$X = < x1, x2, x3, ,,,,, xm]$$

$$Y = < y1, y2, y3 ,,,,, ym]$$

where < x1, x2, x3, ,,,,, xm] is a representation of X as a row vector in the signal space.

1.4 Signal Spaces for communications

For communication we have a desired signal Sd to receive and regenerate at the receiver. There are also other signals S1,S2,,,,Sm generated by different sources that leak into the receive circuit causing interferences. The natures or even number m of the interferences are unknown at the receiver.

The signal space for communications is formed as

$\{Sd, Sin\} = \{ \{Sd\}, \{S1,S2,,,,Sm\}\} = \{Sd, S1,S2,,,,Sm\}$

$Sin = \{S1,S2,,,,Sm\}$ is the subspace formed by the interferences signals.

In communication systems we set a main path receiver that gives signal X and a number of auxiliary paths receivers that give signals Y1,Y2,,,Yn for interferences cancellation, gain enhancement, MIMO and other purposes.

The main path and auxiliary paths signals are expressed as follows;

$X = Sd + I1 \cdot S1 + I2 \cdot S2 +,,,,+ Im \cdot Sm$

$Yi = Di \cdot Sd + Li1 \cdot S1 + Li2 \cdot S2 +,,,, + Lim \cdot Sm$

$(i = 1,2,,,,,n)$

where Sd and $\{Sj ; j = 1,2,,,m\}$ are original signals. Without loss of generality we assume the norms of original signals are normalized: $\| S \| =1$.

The $\{Ij, Lij: i=1,2,,,n, j=1,2,,,,m\}$ are transmission coefficients of the communication paths.

In vector representation X and {Yi} are expressed as

$X = < 1, \quad I1, \quad I2, ,,,, \quad Im]$

$Yi = < Di, Li1, Li2,,,,, Lim] \qquad (i = 1,2,,,,,n \quad Lii =1)$

as vectors in the signal space $\{Sd, S1, S2, ,,,,, Sm\}$.

Signal to Interferences power ratio (SIR)

Suppose we have a single interference signal Si.

Then we need to have only two receivers X and Y;

$X = Sd + I.Si = < 1, I]$

$Y = D.Sd + Si = < D, 1]$

Let us denote Signal to Interferences power ratios (SIR) for X and Y by SIX and SIY respectively. Then

$SIX = \| Sd \|^2 / \| I.Si \|^2 = 1 / |I|^2$

$SIY = \| Si \|^2 / \| D.Sd \|^2 = 1 / |D|^2$

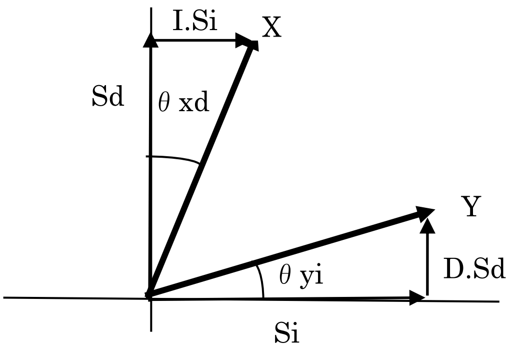
On the other hand in the signal space {Sd, Si} representation

$SIX = 1 / |I|^2 = 1 / \tan^2(\theta xd)$

$SIY = 1 / |D|^2 = 1 / \tan^2(\theta yi)$

The θxd, θyi are respectively the angles between X and Sd and between Y and Si.

The physical meaning of the above definitions will be clear in the following figure.



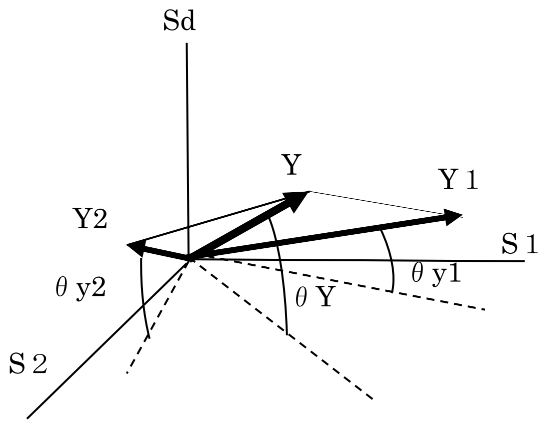
1.5　Representative vector

Suppose we have two interferences S1 and S2. Then we need two auxiliary paths signals Y1 and Y2;

Y1 = D1・Sd + S1 + L12・S2 = < D1, 1, L12]

Y2 = D2・Sd + L21.S1 + S2 = < D2, L21, 1]

The signals S1 and S2 forms a signal space {S1, S2} as a two dimensional plane as shown in the following figure.



Subspace spanned by vectors

The angles θy1 and θy2 depicted in the figure have the following physical meanings;

$\tan^2(\theta y_i) = \| D_i \cdot Sd \|^2 / \| Y_i - D_i \cdot Sd \|^2$　　(i = 1,2)

The linear combination of Y1 and Y2 gives a vector Y on the plane spanned by Y1 and Y2. Then tan^2(θy) for vector Y is defined in the same manner as for Y1 and Y2.

Of the vector Y, there must be at least one vector that gives the maximum tan^2(θy). We name it the **representative vector** and denote it by Y(1,2).

The above concept can be generalized to cases with more vectors {Y1,Y2,,,, Yn} in signal space {Sd,S1,S2,,,,Sm} with greater dimensions..

The representative vector Y(1,2) represents the plane spanned by Y1 and Y2 maximizing the square tangent value against the sub-plane; Sin = {S1,S2,S3,,,Sm}.

In the same manner we can form the representative vector Y(1,2,3) representing the plane spanned by Y3 and Y(1,2).

The procedure continues until we get the representative vector Y(1,2,3,,,n) that represents the plane spanned by Y1,Y2,,,Yn in the signal space {Sd,S1,S2,,,,Sm}.

1.6 Tangent Square Summation theorem

We will now try to get the square tangent value of the representative vector Y(1,2,,,n). The auxiliary paths signals are

$$Yi = Di \cdot Sd + Li1 \cdot S1 + Li2 \cdot S2 +,,,, + Lim \cdot Sm \qquad (i = 1,2,,,,,n)$$

Case of ideal auxiliary paths receivers;

We first analyze an ideal case that the number of the auxiliary receivers is the same as the number of interferences signals; n = m, and each auxiliary path picks purely the targeted interference signal.

$$Yi = Di \cdot Sd + Si \qquad (i = 1,2,,,,,m)$$

The subspace spanned by the auxiliary path signals is

$$Y = [i=1,m]\Sigma\, wi \cdot Yi$$

$$= [i=1,m]\Sigma\, wi \cdot Di).Sd \quad + \quad [i=1,m]\Sigma\, wi \cdot Si$$

The tangent square of Y is given by

$$\tan^2(\theta Y) = |\, [i]\Sigma\, wi \cdot Di \,|^2 \,/\, (\, [i]\Sigma\, |\, wi \,|^2 \,)$$

(To be maximized by wi; i = 1,2,,,,m)

By setting

$$\partial / \partial wi^* = 0 \qquad (i=1,2,,,,m)$$

We get

$$Di^* \cdot (\, [i]\Sigma\, |\, wi \,|^2 \,) - wi \cdot (\, [i]\Sigma\, wi \cdot Di) = 0$$

Or

$$wi / Di^* = (\, [i]\Sigma\, |\, wi \,|^2 \,) / (\, [i]\Sigma\, wi \cdot Di) \qquad (\, i = 1,2,,,,m)$$

They must be all equal to a common value, say K

$$wi / Di^* = K \qquad (i=1,2,,,,m)$$

Which gives

$$\tan^2(\theta Y) = [i=1,m]\Sigma\, |\, Di \,|^2 = [i=1,m]\Sigma\, \tan^2(\theta Yi)$$

Note

$$\tan^2(\theta Y_i) = \| D_i \cdot Sd \|^2 / \| S_i \|^2 = | D_i |^2$$

The objective of the auxiliary paths receivers is to collect the interference signals, hence the leakage of the desired signal component Sd therein is undesired. Therefore the signal to interferences power ratio is inverse of the above tangent values. Thus the above tangent square summation theorem can be restated as inverse SIR summation problem.

$$1/SIY = \tan^2(\theta Y) = \sum_{i=1}^{m} \tan^2(\theta Y_i) = \sum_{i=1}^{m} 1/SIY_i$$

Cases in general;

We have the following situation;

$$Y_i = D_i \cdot Sd + L_{i1} \cdot S_1 + L_{i2} \cdot S_2 +,,,, + L_{im} \cdot S_m \qquad (i = 1,2,,,,,n)$$

In vector forms;

$$[Y> = [D> \cdot Sd + [L][S>$$

Where [Y>, [D>, [S> are column vectors the i-th elements of which are respectively Yi, Di, Si. And [L] is the matrix whose (i,m) component is Lim.

If [L] is regular, the above equation is applied with the inverse matrix [/L],

$$[Y'> = [/L] \cdot [Y> = [D'> \cdot Sd + [S>$$

Where

$$[/L] \cdot [D> = [D'> \qquad \text{Or} \qquad [L] \cdot [D'> = [D>$$

The above situation is now the same as the special case which tells;

$$\tan^2(\theta Y') = \sum_{i=1}^{m} | D_i' |^2 = \sum_{i=1}^{m} \tan^2(\theta Y_i')$$

The above operations are **linear** combinations of the auxiliary paths vectors, which do not alter the structure of the signal subspace {Yi'} = {Yi}, hence

$$\tan^2(\theta Y) = \tan^2(\theta Y')$$

2. Interferences Cancellation by LMSO method

2.1 Least Mean Square Output method

We have a main path circuit X to receive the desired signal Sd, but the output of X also have leakages of interferences signals S1,S2,,,,Sm coming from other sources.
In order to cancel those interferences, we set a number of auxiliary paths receivers; Y1,Y2,,,Yn to get replicas of those interferences signals.

The main path and auxiliary paths signals are combinations of those signals;

X = Sd + I1・S1 + I2 ・ S2 +,,,,+ Im ・ Sm

Yi = Di ・ Sd + Li1 ・ S1+Li2 ・ S2 +,,,, + Lim ・ Sm (i = 1,2,,,,,n)

The {Di, Ii, Lij: i=1,2,,,n, j=1,2,,,,m} are transmission coefficients of the communication paths.

Least Mean Square Output Method (LMSO)

In order to cancel the interference signals, we subtract a combination of the auxiliary paths signals with adaptive weights to get the compensated signal Z.

Z = X – [i=1,n] Σ Wi ・ Yi

where {Wi; I = 1,2,,,,n} are the adaptive weights to be controlled adaptively.

Design philosophy of LMSO method

The **power** of output signal Z is assumed to get **minimal** if the interferences signals are successfully cancelled.

We control the weights Wi (I = 1,2,,,,n) to minimize ‖ Z ‖ ^2.
For the **necessary** condition we set the partial derivatives of ‖ Z ‖ ^2 by Wi* to zero.

∂ ‖ Z ‖ ^2 / ∂ Wi* = 0

Then we get;

(Z, Yi) = 0 (i = 1,2,,,,n)

That is, the output signal must be orthogonal to all the auxiliary paths signals.

The weights {Wi} can be derived from the equation.

[k=1,n] Σ (Yk, Yi) ・ Wk = (X, Yi) (i = 1,2,,,,n)

The equations can be expressed more simply;

[(Yk,Yi)] ・ [Wk> = [(X,Yi)> (k, i = 1,2,,,, n)

where [(Yk,Yi)] is an n x n matrix with (Yk,Yi) as its (i,k) elements and [Wk> a column (vertical) vector with Wk as the k-th element.

Note the [(Yk,Yi)] is an Hermite matrix;   [(Yk,Yi)] = [(Yi,Yk)]*

2.2. Signal Space Analysis of LMSO methods

As the output Z must be orthogonal to all auxiliary paths signals Y1,Y2,,,,Yn, Z must be orthogonal to the representative vector Y(1,2,,,,n) representing the subspace {Y1,Y2,,,Yn}.

For simplicity let us denote Y = Y(1,2,,,,Yn), which has the following composition;

$$Y = Dy.Sd + Sy \qquad (Sy \text{ normalized;} \quad \| Sy \| = 1)$$

where Sy is a linear combination of the interferences signals S1,S2,,,,Sm.

The output signal Z must be orthogonal to Y, hence must have the following composition;

$$Z = Sd + Iy.Sy$$

and

$$(Z, Y) = Dy^* + Iy = 0$$

The SIR for Z and Y are

$$SIZ = \| Sd \| \hat{} 2 / \| Iy.Sy \| \hat{} 2 = 1 / |Iy| \hat{} 2$$

$$SIY = \| Sy \| \hat{} 2 / \| Dy.Sd \| \hat{} 2 = 1 / |Dy| \hat{} 2$$

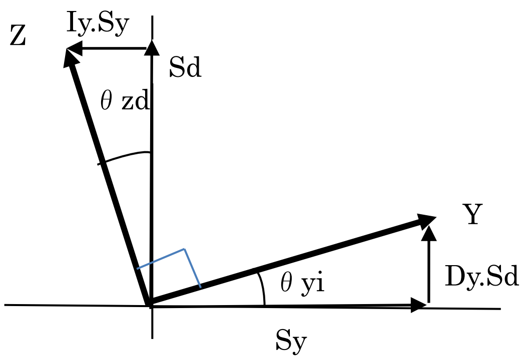
Hence

$$SIZ = SIY$$

In angles representation;

$$1/ \tan\hat{}2(\theta zd) = 1 / \tan\hat{}2(\theta yi)$$

The orthogonality of Z and Y is depicted in the following figure.



2.3 Problems of LMSO methods

[1] Performances degradation

The analysis above tells the SIR of the output Z is equal to that of representative vector Y of the auxiliary paths signals subspace regardless of that of the main path X. In most situations the SIR of the main path is higher thus the LMSO operations degrade rather than improve the SIR performances.

[2] Trivial zero output problem

The tangent square summation (TSS) theorem tells

(1) The SIR of the auxiliary subspace monotonically degrades as the number of auxiliary path signals increases.
(2) If the number of the auxiliary paths gets larger than the number of the interferences signals in the system, then the output of the interferences cancellation circuit must trivially be zero.
(3) The mechanism of the problem is evident from the signal space theory. Controlled by LMSO method, the output Z must be orthogonal to all auxiliary path signals Y1.Y2,,,Yn in the signal space which is m-dimensional. If $n > m$, there can be no non-zero vector Z orthogonal to all auxiliary paths signals; more vectors than the dimension of the signal space.

2.4. Effects of thermal noise

We now analyze effects of thermal noise for the LMSO operations.

[1] Noise in Signal Space

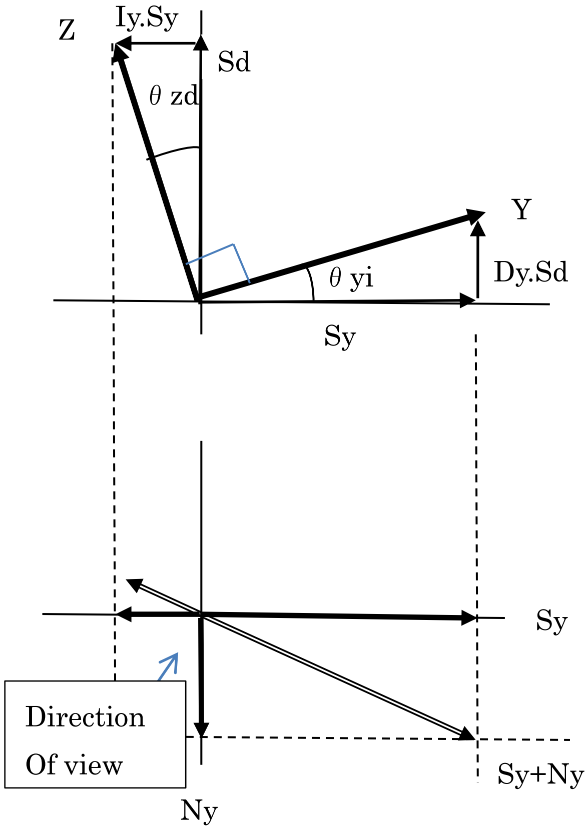
Thermal noise is non-coherent and orthogonal to any other signals or noises. In communication networks the noises are band-limited which gives the noises finite auto-correlation properties for finite time differences. In short, they act like a randomly modulated signal hence can be accommodated into the signal space theory.

The representative vector Y is now modified to include thermal noise Ny;

$$Y = Dy.Sd + Sy + Ny \qquad (Sy \text{ normalized;} \quad \| Sy \| = 1)$$

The TSS theorem tells the angle of Y against {Sy,Ny} does not change from that of Y-Ny against Sy. But the direction of the vector Y - Dy.Sd changes to Sy + Ny from Sy alone. The output Z in the signal space also changes since it must be now orthogonal to Y rather than Y – Ny.

The above change is depicted in the following figure.



The effect of thermal noise is now analyzed in equations.

The receive signals now contain thermal noises..

$$X = Sd + I1 \cdot S1 + I2 \cdot S2 +,,,,+ Im \cdot Sm + Nx$$

$$Yi = Di \cdot Sd + Li1 \cdot S1 + Li2 \cdot S2 +,,,, + Lim \cdot Sm + Ni \qquad (i = 1,2,,,,,n)$$

The output Z

$$Z = X - \langle Wi][Yi\rangle$$

is controlled to be orthogonal to all Yi (i = 1,2,,,,n)

Then the adaptive weights must meet the following equation.

$$[(Yk,Yi)] \cdot [Wk\rangle = [(X,Yi)\rangle \qquad (k, i = 1,2,,,, n)$$

Because the thermal noises Ni (i = x, 1,2,,,,n) are uncorrelated with any other signals or noises than themselves, the right hand side of the above equation [(X,Yi)> remain the same regardless of the noises.

The coefficients [(Yk,Yi)] remain the same except for the diagonal components.

In the case the noise power is the same for all auxiliary paths,

$$\| N_i \|^2 = \| N \|^2 \qquad (i=1,2,,,n)$$

Then the above equation changes to

$$\{ [(Y_k', Y_i')] + \| N \|^2 . [I] \} \cdot [W_k> = [(X, Y_i')> \qquad (k, i = 1,2,,,, n)$$

and

$$[(Y_k', Y_i')] \cdot [W_k'> = [(X, Y_i')> \qquad (k, i = 1,2,,,, n)$$

where [I] is an identity matrix and the primed (') symbols mean the parameters in the case of no thermal noises, i.e. $Y_i' = Y_i - N_i$.

In the case of weak noise $\| N \| << 1$,

$$[W_k > = (1 - \| N \|^2) [W_k' >$$

Then the output Z is

$$Z = X - <W_k'][Y_k'> + (\| N \|^2).<W_k'][Y_k'> - (1 - \| N \|^2). <W_k'][N_k>$$

(Noiseless case) (control error by noise) (additive noise)

Stabilizer effects of thermal noise

Thermal noises expands the dimension of the signal space from m+1 to m+1+n where m, n are respectively the number of original signals and that of auxiliary paths. As the dimension of the signal space gets greater than that of the auxiliary paths the trivial zero problem is avoided.

3. Improved LMSE method

3.1 Gaussian Least Mean Square Error method

The problems of LMSO method are caused by simply minimizing the power of the output of the interferences cancellation circuit.

In order to solve the problems, the Gaussian least-mean-square-error method is now applied to the system.

Principle of Gaussian LMSE method

(1) Generate candidate replicas of the desired signal based on the receive data.
(2) Calculate summation of the square errors which are differences between the desired signal replicas and the receive data
(3) Adopt the replica that minimizes the summation of the square errors.

3.2. Interferences cancellation by LMSE method

In order to get the **errors** of the desired signal, we need to regenerate a **replica of the desired signal Sd** at the receiver, which is the very objective of communication.
In digital communications the desired signal can be regenerated at the receiver by demodulation with a good likelihood if the SIR and SNR are sufficiently high. Then the regenerated desired signal replica can be used to remove the desired signal component in the correlation measurement. This method is also called **decision-feedback**, has been widely used in digital communications.

A simple analysis follows to show the mechanism of the improvement. The symbols <A], [B>, [C] respectively stand for the row vector, column vector and matrix.

$$X \;=\; Sd \;+ <I\,] \cdot [S>$$

$$[Y> \;=\; [D> \cdot Sd \;+ [L] \cdot [S>$$

Then the canceller output is

$$Z = X - <W] \cdot [Y> \;=\; (1 - <W \cdot D>)\, Sd \;+ (<I) - <W \cdot L]) \cdot [S>$$

From Z we regenerate a replica of the desired signal Sd' and subtract it from Z.
Let

$$Sd - Sd' = \varepsilon \,.\, Sd \qquad (\,|\,\varepsilon\,| << 1\,)$$

Then we get

$$Z' = (1 - <W \cdot D>)\, \varepsilon\, Sd \;+ (I - <W \cdot L]) \cdot [S>$$

Now the correlation measurement is made between Z' and Y to control the adaptive weights {Wi} to achieve

(Z', Y) = 0

Let

[Y'> = [D> ・ ε Sd + [L] ・[S>

Then, the following equivalence relation holds **mathematically** in the correlation measurement

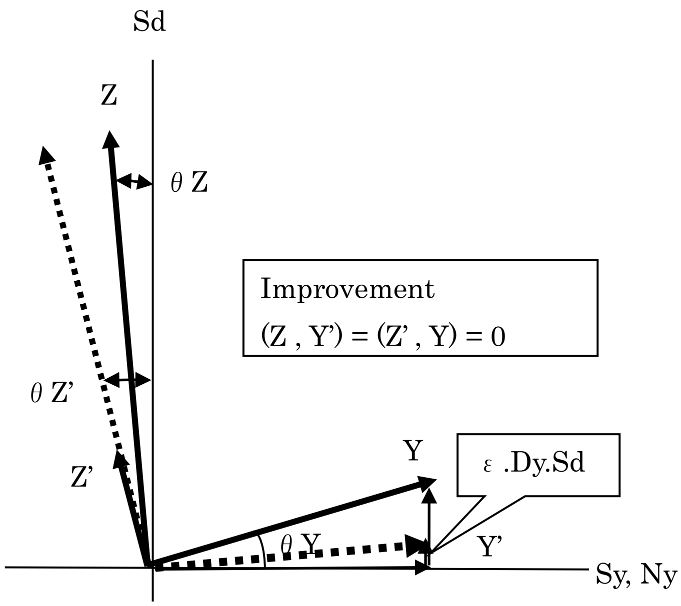
(Z', Y) = (Z, Y')

Thus SIR of the output Z be improved by 1/ ε ^2 times.

SIZ = SIY' = SIY / ε ˆ2

The mechanism of the improvement is depicted in the following figure.

Note in the above equation (Z', Y) is a real measurement and (Z, Y') is purely mathematical since [Y'> is only virtual.



3.3. Methods of desired signal suppression

3.3.1. Demodulation for digital modulation

In digital communications the desired signal replica is regenerated by demodulation of the signal. In this case ε is approximately equal to the bit error rate (BER), which is usually very small. Thus a very great improvement can be achieved by the proposed method. The method is generally called "**decision feedback equalizer**" and has been widely used.

3.3.2. Hard limiting for general modulation

In general communications including analog modulations regenerating accurate replicas of the desired signals in interference environment is not easy. A useful method in such situations is hard limiting if the initial condition is met that the power of the desired signal is sufficiently greater than that of the interferences signals.

Let us now analyze how hard limiting works on a desired signal and an interference signal. Let a signal be

$$Z = A \cdot \cos(\omega c\,.t) + a.\cos(\omega 1\,.t + \varphi)$$

The first and second terms are respectively the desired and the interference signals. In general they are different in frequency and phase.

By setting

$$\Theta = (\omega 1 - \omega c).t + \varphi$$

we can rewrite

$$Z = (A + a.\cos(\Theta)).\cos(\omega c.t) \;-\; a.\sin(\Theta).\sin(\omega c.t)$$

If A >> a, the in-phase component a.cos (Θ) is cancelled in the hard limiter which gives the output;

$$Zh\ (=)\ A.\cos(\omega c.t) - a.\sin(\Theta).\sin(\omega c.t)$$
$$(=)\ A.\cos(\omega c.t - \Phi)$$

where

$$\Phi = \arctan(a/A.\sin(\Theta))$$

and (=) means nearly equal.

The mechanism of the hard limiting is depicted in the following phasor diagram.

The hard limiter output approximately is

$$Zh\ (=)\ Z \;-\; a.\cos(\Theta).\cos(\omega c.t)$$
$$= A.\cos(\omega c.t) + a/2.\cos((2\omega c - \omega 1).t - \varphi) \;-\; a/2.\cos(\omega 1).t + \varphi)$$

We observe the following points about Zh(t);

(1) The amplitude of the interference signal is halved.

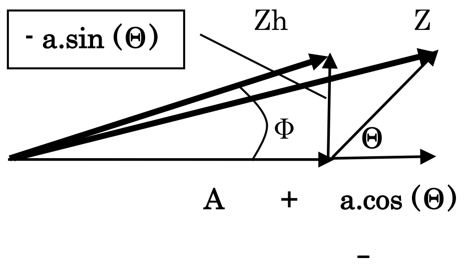
(2) A mirror image of the interference signal against the desired signal appears with the same amplitude as the interference signal.

(3) The desired signal, the interference signal and the mirror image of the interference signal are mutually uncorrelated or orthogonal in the signal space.

(4) The amplitudes of the interference and the generated mirror image signals are halved to reduce the power to 1/4 of the original value. Hence the SIR of the output of the hard limiter is improved by 3dB in total and 6dB against the interference

signal itself. This phenomenon is an instance of **small signal suppression effect** in non-linear devices [3].

A phasor diagram on hard limiting is given in the following.



Phasor diagram of Hard Limiting

Desired signal reduction

We subtract Zh with adaptive weight V from Z to get Z'.

$$Z' = Z - V.Zh$$

The Z' is controlled to be orthogonal to Zh;

$$(Z', Zh) = 0$$

From the equation we get

$$V = (1+ \delta^2/4) / (1+ \delta^2/2)$$

where $\delta = a/A$

If $\delta << 1$, then

$$Z' \propto \delta^2/2 \cdot A\cos(\omega c.t) + a.\cos(\omega 1.t + \varphi) - a/3.\cos((2\omega c - \omega 1).t - \varphi)$$

Thus the desired signal is reduced by the factor $(a/A)^2/2$ which will improve the performance of the correlation measurements between Z' and the auxiliary paths signals {Yi i=1,2,,,n}.

4. Circuits and Operations

We have at the receiver, the main path signal X and the auxiliary signals {Yi; I = 1,2,,,,n}. We try to cancel the interferences signals in X by subtracting Yi multiplied with adaptive weight Wi to get the output Z.

$$Z = X - [i=1,n]\,\Sigma\, Wi \cdot Yi$$

From Z we regenerate the desired signal replica Sd' and subtract the element from Z by LMSE method to get Z'.

We then determine the weight Wi (I = 1,2,,,,n) by the orthogonalization condition;

$$(Z', Yi) = 0 \quad (i = 1,2,,,,n)$$

4.1. Time sections for control

We conduct the above processing in successive time sections {Ts.n ; n= 0,1,2,3,,,,,,}. The time length of each section Ts needs to be selected to achieve sufficiently accurate time averaging in each section. We denote the variables in the n-th time section by added [n] as follows;

$$(Z', Yi)[n] = [t= n.Ts\ ,\ (n+1).Ts] \int Z'(t) \cdot Yi^*(t)dt$$

4.2. Adaptive control of weights {Wi}

We control the adaptive weights Wi by the following difference equation;

$$Wi[n] = Wi[n-1] + g.\ (Z', Yi)[n-1] / (Yi,Yi)$$

$$Z(t)[n] = X(t)[n - [i=1,n]\,\Sigma\, Wi[n] \cdot Yi(t)[n]$$

where g is the loop gain of the interferences signal cancellation loop..

The steady state Wi[n] = Wi[n-1], is achieved when the orthogonality is completed;

$$(Z', Yi)[n] = 0$$

4.3. Desired signal elimination

Let the regenerated desired signal in time section n by Sd'(t)[n], then the desired signal is eliminated from the output signal Z[n] by the formula;

$$Z'(t)[n] = Z(t)[n] - V[n-1].\ Sd'(t)[n]$$

$$V(n) = V(n-1) + g' \cdot (Z', Sd')[n-1] / (Sd',Sd')$$

where g' is the loop gain of the Desired signal elimination loop. The steady state V[n] = V[n-1], is achieved when the orthogonality is completed; (Z',Sd')[n] = 0

4.4. Loop stability

The stability of the interferences cancellation loops now needs to be examined. As the desired signal elimination is made by the same algorithm, we will examine only the interferences cancelation function in the following analysis.

What to be checked are ;

$Wi[n]- Wi[n-1] = g.(Z, Yi)[n-1] /(Yi,Yi) \quad (i = 1,2,,,n)$

$Z[n] = X[n] - [i=1,n] \Sigma Wi[n].Yi[n]$

The above two equations are joined to give;

$Wi[n] - Wi[n-1]$

$= g.\{ (X, Yi)[n-1] /(Yi,Yi) - [j=1,n] \Sigma Wj[n-1].(Yj,Yi)/(Yi,Yi) \}$

$= g.\{ \alpha i - [j=1,n] \Sigma Wj[n-1]. \beta ji / \beta ii \}$

where

$\alpha i = (X, Yi) /(Yi,Yi)$

$\beta ji = (Yj,Yi) /(Yi,Yi) \quad (i,j = 1,2,,,,n)$

Note αi, βji are stationary in time hence nearly constant for sufficiently large time interval Ts.

In Z-transformation

$(1- z^{-1}).Wi(z) = g.\alpha i / (1- z^{-1}) - [j=1,n] \Sigma Wj(z).z^{-1}. \beta ji / \beta ii \qquad (i= 1,2,,,n)$

where

$W(z) = [n=0,\infty] \Sigma W[n].z^{-n}$

The above equation is modified;

$[j=1,n] \Sigma \{ (1-z^{-1}) \delta ji +g. \beta ji/ \beta ii. z^{-1}\}.Wj(z) = g.\alpha i / (1-z^{-1})$

In vector and matrix format;

$<Wj(z)].[(1-z^{-1})\delta ji +g.\beta ji /\beta ii. z^{-1}] = g./ (1-z^{-1})<\alpha i]$

Where δji is Dirac delta function and $<xj]$ is row vector with xj in the j-th element and [xji] is matrix with xji as (j,i) element.

4.4.1. A single auxiliary path

The above equation reduces to

$[1- (1- g.) z^{-1}].W(z) =g.\alpha/ (1-z^{-1})$

Or

$W(z) = g. \alpha .z^2 / \{(z-1) \cdot (z- (1- g))\}$

The n-th output is obtained by the inverse z-transform;

$W[n] = 1/ (2 \pi i).[| z | =1] \int W(z).z^{(n-1)} dz = \alpha/\beta.\{ 1 - (1- g)^{(n+1)} \}$

The stability condition is

$| 1- g | < 1 \quad ,or \quad 0 < g < 2$

The correlation error exponentially converges to zero.

$(Z[n], Y) = (X,Y).(1-g)^n$

4.4.2. Cases of two auxiliary paths

$$[(1-z^{-1})\delta_{ij} + g.\beta_{ij} / \beta_{ii}.\, z^{-1}][W_j(z)> \quad = g./\, (1-z^{-1})[\alpha_i> \qquad (i,j = 1,2)$$

Or

$$\begin{bmatrix} 1-(1-g).z^{-1} & g.\beta_{12} / \beta_{11}.z^{-1} \\ g.\beta_{21}/\beta_{22}.z^{-1} & 1-(1-g).z^{-1}] \end{bmatrix} \begin{bmatrix} W1 \\ W2 \end{bmatrix} = g / (1-z^{-1}) \begin{bmatrix} \alpha 1 \\ \alpha 2 \end{bmatrix}$$

where the boxes denote matrix or vectors.

The stability condition is the characteristic roots of the equation

$$\det \begin{bmatrix} 1-(1-g).z^{-1} & g.\beta_{12} / \beta_{11}.z^{-1} \\ g.\beta_{21}/\beta_{22}.z^{-1} & 1-(1-g).z^{-1}] \end{bmatrix} = 0$$

where det means determinant of the matrix following.

It can be calculated to give the characteristic roots r1, r2;

$$r1 = 1 - g + g.\,|\beta_{12}| / \sqrt{(\beta_{11}.\beta_{22})}$$

$$r2 = 1 - g - g.\,|\beta_{12}| / \sqrt{(\beta_{11}.\beta_{22})}$$

From the condition that the absolute values of r1 and r2 be smaller than 1, we get the stability condition

$$0 < g < 2 / (1 + |(Y1,Y2)| / (\| Y1 \| . \| Y2 \|)$$

Note $|(Y1,Y2)| / (\| Y1 \| . \| Y2 \|)$ is the likelihood between Y1 and Y2,which equals to 1 if Y1 and Y2 are identical and zero if they are uncorrelated. The stability condition for the case of two auxiliary paths signals is the same as for the case of a single auxiliary path signal if the two auxiliary paths signals are mutually orthogonal, because then the two cancellation loops function independently.

4.4.3. Arbitrary number of auxiliary paths

In general the cancellation loops with n auxiliary paths ($n > 2$) signals cases will function stably if the auxiliary path signals are highly independent and the loop gain g is set at sufficiently small values.

If the initial auxiliary paths signals {Y1,Y2,,,,Yn} are first transformed to an orthogonal sets of auxiliary paths signals {Y1", Y2",,,,Yn"} by an orthogonalizing procedure, then each adaptive weight Wi (i=1,2,,,n) can be controlled independently.

For the orthogonalizing procedure, Schmit's orthogonalizing method or eigen function method on [(Yk,Yi)] etc. are available as standard linear algebraic algorithms.

5. Applications

The interferences cancellation technologies have been applied to wide ranges of applications.

5.5.1. Channel equalizers for digital signal transmission

The inter-symbol interferences occur by channels fading or equipment faults such as channel filters mismatches or errors in symbol timing recovery circuits. The main path is the symbols at data decision timing and the auxiliary paths signals are at symbol timings in the past and future around the decision timing. The decision-feedback equalizer is an exact implementation of the interferences cancellation as described in this paper.

5.5.2. Echo cancellation

The echoes occur by the reflection of the voice signal at the far end of the transmission lines. An exact replica of the interference is readily available at the sender as delayed version of the transmit signal hence can be fully cancelled by a simple interference cancellation.

5.5.3. Dual polarization radio wave transmission system

Dual polarizations of radio waves can readily double the channel capacity with the same frequency bandwidth.

Let the receive signal be

$$Y1 = L11.S1 + L12.S2$$

$$Y2 = L21.S1 + L22.S2$$

Here both S1 and S2 are desired signals and interferences signals.

In order to cancel mutual interferences we conduct

$$Z1 = Y1 - W1.Y2 = (L11 - W1.L21).S1 + (L12 - W1.L22).S2$$

$$Z2 = Y2 - W2.Y1 = (L21 - W2.L11).S1 + (L22 - W2.L12).S2$$

The exact solutions are

$$W1 = L12 / L22$$

$$W2 = L21 / L11$$

which perfectly regenerate the original signals.

In order to get those transmission links parameters pilot signals are inserted with the signal transmitter or beacons from the satellites are utilized [4].

In this paper we will study the methods that can work without pilot signals.
From Z1, Z2 we regenerate replicas of S1, S2 denoted as S1' and S2'.

[1] Demodulator methods

In digital communications good replicas of the desired signals can be regenerated at the receiver.

$$S1' = \sqrt{(1-\varepsilon^2)}.S1 + S1''$$

$$S2' = \sqrt{(1-\varepsilon^2)}.S2 + S2''$$

The S1" and S2" are errors generated in the desired signal regeneration processes. The norm of S1" is

$$\| S1'' \|^2 = \varepsilon^2$$

so

$$\| S1' \|^2 = \| S1 \|^2 = 1$$

$$\| S2' \|^2 = \| S2 \|^2 = 1$$

In digital communications the error rate ε^2 is roughly the symbol error rate at the demodulator.

Note S1", S2" are uncorrelated with any other signals as they are randomly generated.

By LMSE we achieve

$$(Z1, S2') = (Z2, Si') = 0$$

Then we get

$$W1 = (Y1, S2') / (Y2, S2') = L12 / L22$$

$$W2 = (Y2, S1') / (Y1, S1') = L21 / L11$$

which are the exact solutions.

Thus we can expect to realize accurate dual polarization signal transmission radio systems.

[2] Hard limiter methods

For the input

$$Z1 \propto S1 + a.S2 \quad (|a| < 1)$$

the output of the hard limiter is

$$Z1h \propto S1' = S1 + \delta.a.S2 + \delta.a.S1'' \qquad (|\delta| < 1)$$

S1"is the mirror image of S2 against S1. Note S1"is orthogonal to both S1 and S2.

Likewise for Z2;

$$Z2 \propto S2 + a.S1 \quad (|a| < 1)$$

the output of the hard limiter is

$$Z2h \propto S2' = S2 + \delta.a.S1 + \delta.a.S2'' \quad (|\delta| < 1)$$

By LMSE function Z1 is made orthogonal to S2'and Z2 to S1'. By hard limiter functions S2' is produced from Z2 and S1' from Z1 with improved SIR.

Thus we have the following cycles.

Z1 — improvement → S1'

S1'—orthogonal → Z2

Z2 --- improvement → S2'

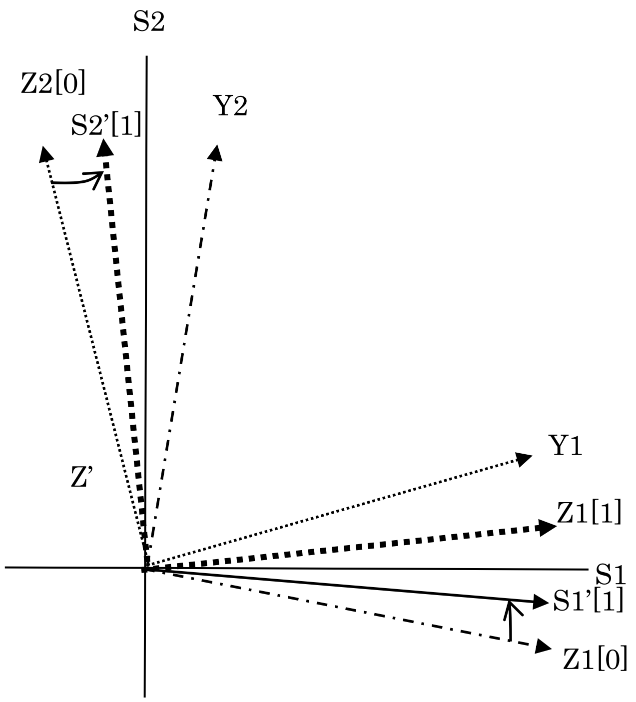
S2' —orthogonal → Z1

The above transitions repeat the cycle of improvement until a nearly complete compensation of the cross polarization interferences is achieved.

In the initial phase Z1[0], Z2[0] are respectively made orthogonal to Y2 and Y1.

Then the hard limiters produce S1'[1] and S2'[1] respectively from Z1[0] and Z2[0]. The loops then function to make Z1[1] and Z2[1] respectively orthogonal to S2'[1] and S1'[1]. The above procedure continues endlessly. In each step generation of S1'[n], S2'[n] respectively from Z1[n-1], Z2[n-1] the SIR are improved, S1' and S2' approach to S1 and S2 coordinates. Thus the above improvement process repeats itself until it comes to the limits caused by thermal noise.

The above process is depicted in the following figure.



5.5.4. Multiple Inputs Multiple Outputs (MIMO)

The above dual polarization mode communication system can be readily generalized to MIMO systems with larger numbers of the signals and receivers.

The conventional MIMO system was based on the orthogonalization of the receive signals by eigen-vectors methods making use of the Hermitian nature of the correlation matrix of the receive signals [5]. The orthogonalization alone is insufficient for MIMO function because the **originality** of those signals are not regenerated or enhanced.

In the herein proposed system the SIR improvement is achieved by the use of small signals suppression effect of non-linear operations such as demodulation or hard-limiting.

5.5.5. Satellites Systems Interferences Cancellation

The method described herein is readily applicable to solve those interferences problems as adjacent satellites, inter-beams or interferences with terrestrial communications networks.

5.5.6. Cellular systems Interferences Cancellation

- Inter-cells interferences at the mobile
- inter-cells, inter-sectors or with external systems interferences at a base station

5.5.7. Noise cancellers

The method presented in this paper is applicable to wide ranges of applications so long as the main path and auxiliary paths signals are available with significant independence. A pri.o.ri knowledge about the desired signal is useful to regenerate a good replica of the desired signal which can be used to generate the errors that is to be minimized by LMSE algorithms.

6. Conclusion

The signal space analysis proposed in the previous papers [1,2] was restated and applied to general cases including external interferences and thermal noises.

The function of interferences cancellation system was analyzed on concrete models to establish the stability conditions of the loops.

The function of hard limiter as a device for generation of the desired signal replica was analyzed.

The function of the dual polarization radio communication system is analyzed on two different methods; demodulation and hard limiting for generation of the desired signal replica

The methods are applicable to general MIMO (Multi-Input-Multi-Output) system with more than 2 signals.

The methods proposed in the paper are fundamental and applicable to wide ranges of applications.

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