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A Signal Space Analysis of Interferences Cancellation Systems

- Communication is recovery of Originality rather than Orthogonality of Receive Signals –

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Human Network for Better 21 Century

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Abstract

Interferences among signals from different sources are universal problems in communication networks. Typical examples are interferences coming through antenna side-lobes in radio networks. Daily interferences come through power lines from engines of automobiles, airplanes or other sources. The inter-symbol interferences in digital communications and echoes in long distance cables systems can be modeled as the interference systems despite the facts that they are generated by the same sources. In some cases interference systems are positively utilized for communications in order to increase the channel capacity. A classic example is reuses of orthogonal polarizations of radio waves that can readily double the channel capacity. Other examples are CDMA (Code-Division-Multiple Access) that can efficiently reuse frequency resources and MIMO (Multiple-Input-Multiple-Output) that can efficiently reuse the space resources. In some cases the interferences signals come from hostile sources as in military applications. Those versatile interferences systems can be analyzed by a signal space theory in a unified form.

The signal space is a multi-dimensional Hilbert space formed by signals originating from independent sources. Signals from different sources are uncorrelated and form orthogonal bases of the signal space. As the receive signals are combinations of signals from a number of different sources, they can be represented by vectors in the signal space. Then the interferences systems can be statically expressed as vectors in the signal space, thus a simple and unified analysis of varieties of communication networks becomes possible. It is important to discriminate originality and orthogonality of signals. Signals from different sources are uncorrelated, that is, they are inherently mutually orthogonal. In interferences cancellation systems a number of auxiliary receivers are equipped in addition to the main receiver. The correlations measurements among the receive paths signals give a correlation matrix which is an Hermie matrix that can be orthogonalized by eigen values & vectors methods. Thus we can recover the orthogonality of the receive paths signals. However, it is apparent that the originality of the desired signal is not recovered because the main and auxiliary paths signals are combinations of the original signals from different sources. The objective of communication networks is recovery of original signals, hence recovery of orthogonality of the receive paths signals can not achieve the objective of the communications networks. .

Keywords

Interferences, LMSE, Decision Feedback, Antenna Side-lobe, Correlation, Uncorrelated, Orthogonality, Originality ,

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1. Signals and Signal Space

Fourier Transform;

A time signal s(t) can be also expressed on frequency axis by the Fourier transform.

$$S(j\omega) = [-\infty,+\infty] \int s(t) \cdot e^{(-j\omega t)} \cdot dt$$

The inverse transform is

$$s(t) = 1/(2\pi)\,[-\infty,+\infty] \int S(j\omega) \cdot e^{(j\omega t)} \cdot d\omega$$

where ω is the angular frequency and j the imaginary number unit; $j \cdot j = -1$.

In general the signal s(t) can be a complex function of t.

Inner Product or Correlation;

Suppose we have two signals s1(t) and s2(t). Then we can define the **inner product** of those signals;

$$(s1(t), s2(t)) = [-\infty,+\infty] \int s1(t) \cdot s2^*(t)\, dt$$

Where s2(t)* means the complex conjugate of s2(t)

The inner product can also be defined on the frequency axis;

$$(S1(j\omega), S2(j\omega)) = 1/(2\pi)\,[-\infty,+\infty] \int S1(\omega) \cdot S2(\omega)^* \cdot d\omega$$

It can be shown the above inner products are identical;

$$(s1(t), s2(t)) = (S1(j\omega), S2(j\omega))$$

The above inner products are also called **correlation** of the signals s1(t) and s2(t).

Power of signals;

The self correlation of a signal s(t) is physically the power of the signal;

$$(s(t), s(t)) = [-\infty,+\infty] \int |s(t)|^2\, dt$$
$$= 1/(2\pi)\,[-\infty,+\infty] \int |S(j\omega)|^2 \cdot d\omega$$
$$= (S(j\omega), S(j\omega)) = \| S(j\omega)\|^2$$

where $\| S(j\omega)\|$ is called the **norm** of the signal $S(j\omega)$.

$|S(j\omega)|$ means the absolute value of $S(j\omega)$.

Schwarz inequality;

Suppose we have two signals x(t) and y(t) and their Fourier Transforms $X(j\omega)$ and $Y(j\omega)$. Then Schwarz inequality states;

$$|(X(j\omega), Y(j\omega))| = \text{or} < \|X(j\omega)\| \cdot \|Y(j\omega)\|$$

Likelihood or Angles Between Signals in Signal Space;

The correlation or inner product between two signals X and Y can be expressed as follows;

$$(X, Y) / (\|X\| \cdot \|Y\|) = \cos(\theta) \cdot e^{(j\phi)}$$

where θ is the **angle** between vectors X and Y in the Signal Space and ϕ is the phase of the complex value (X,Y).

The amplitude of the above formula;

$$\cos(\theta) = |(X, Y)| / (\|X\| \cdot \|Y\|)|$$

is also called the **likelihood** of signals X and Y.

For $\theta = 0$, the signals are identical; X = Y or totally correlated.

For $\theta = \pi/2$, $\cos(\theta) = |(X, Y)| / (\|X\| \cdot \|Y\|)| = 0$, the signals X and Y are totally **uncorrelated** or mutually **orthogonal** in the Signal Space.

Signal Space;

Suppose we have signals S1, S2, ,,, Sm from different sources. Then they form a signal space with each signal giving the bases of the space. Without loss of generality, we can normalize their amplitude to 1. $\|Si\|=1$ for all i.

Here we define **originality** and **orthogonally** of the signals. If two signals S1 and S2 are generated from different sources, then they are **original and** mutually **orthogonal.**

The inverse is not necessarily true. For example if we have two signals X and Y;

$$X = S1 + a \cdot S2 \quad \text{and } Y = b \cdot S1 + S2,$$

then

$$(X, Y) = b^* + a = 0 \quad (\text{if } a = -b^*)$$

The signals X and Y can be mutually orthogonal despite the fact they are not original signals from separate sources.

The above signal space is a Hilbert space where the inner product is defined.

It is also a vector space spanned by the original signals {Si ; i = 1,2,3,,,,m}.

Any signal in the communication system is a combination of those signals originating from different sources.

Suppose

$$X = x1 \cdot S1 + x2 \cdot S2 + ,,,\ xm \cdot Sm$$
$$Y = y1 \cdot S1 + y2 \cdot S2 + ,,,\ ym \cdot Sm$$

Then

$$(X,Y) = x1 \cdot y1^* + x2 \cdot y2^* +,,,,\ xm \cdot ym^*$$

Thus the signals X and Y can be expressed as vectors in the signal space;

$X = (x1, x2, x3, ,,,,, xm)$
$Y = (y1, y2, y3 ,,,,, ym)$

The signal space concepts were proposed in 1970s and applied to analyses of inter-symbols and other interferences problems [3], [4].

2. Interference cancellation system

Suppose we have a desired signal Sd to receive and regenerate for communication. There are also other signals S1,S2,,,,Sm generated by different sources that leak into the receive circuit. In order to cancel those interferences, we set a number of auxiliary receivers. Let us denote the main receiver by X that is to receive the desired signal Sd, and the auxiliary receivers Y1,Y2,,,Yn to receive the interferences signals.

The main path and auxiliary paths signals are combinations of those signals;

$X = Sd + I1 \cdot S1 + I2 \cdot S2 +,,,,+ Im \cdot Sm$
$Yi = Di \cdot Sd + Li1 \cdot S1 + Li2 \cdot S2 +,,,, + Lim \cdot Sm$
$(i = 1,2,,,,,n)$

where Sd and {Sj ; j = 1,2,,,m} are original signals. Without loss of generality we assume the norms of original signals are normalized: $||S||=1$.

The {Ii, Lij: i=1,2,,,n, j=1,2,,,,m} are transmit coefficients of the communication paths.

Least Mean Square Error Method (LMSE)

In order to cancel the interference signals, we subtract a combination of the auxiliary paths signals with adaptive weights to get the compensated signal Z.

$Z = X - [i=1,n]\Sigma\, Wi \cdot Yi$

where {Wi; I = 1,2,,,,n} are the adaptive weights.

We assume the power of signal Z will be minimal when the intended interferences cancellation is achieved. We control the weights Wi (I = 1,2,,,,n) to minimize $||Z||^2$.

To do that we set the partial derivatives of $||Z||^2$ by Wi*.

$\partial ||Z||^2 / \partial Wi^* = 0$

Then we get;

$(Z, Yi) = 0 \quad (i = 1,2,,,,n)$

That is, the output signal must be orthogonal to all the auxiliary paths signals.

The weights {Wi} can be derived from the equation.

$[k=1,n]\Sigma\,(Yk, Yi) \cdot Wk = (X, Yi)$
$(i = 1,2,,,,n)$

The equations can be expressed more simply;

$[(Yk,Yi)] \cdot [Wk> = [(X,Yi)> \quad (k, i = 1,2,,,, n)$

where [(Yk,Yi)] is an n x n matrix with (Yk,Yi) as its (k,i) elements and [Wk> a column (vertical) vector with Wk as the k-th element.

Note the [(Yk,Yi)] is an Hermite matrix;

$[(Yk,Yi)] = [(Yi,Yk)]^*$

3. Signal Space Analysis of LMSE Operations

We will now deal with the simplest case where we have only a desired signal Sd and an interference signal S1.

$X = Sd + I \cdot S1$
$Y = D \cdot Sd + L \cdot S1$

Then the canceller output

$Z = X - W \cdot Y$
$= (1 - W \cdot D)Sd + (I - W \cdot L) \cdot S1$

From $(Z, Y) = 0$,

we get

$W = (D^* + L^* \cdot I) / (|D|^2 + |L|^2)$

where we used $||Sd||^2 = ||S1||^2 = 1$.

And the output is

$Z = A \cdot (L^* \cdot Sd - D^* \cdot S1)$

where $A = (L - D \cdot I) / (|D|^2 + |L|^2)$

Let us denote the Signal-to-Interference Power Ratios (**SIR**) of the main, auxiliary and the output signals as

$SIX = ||Sd||^2 / ||I \cdot S1||^2 = 1 / |I|^2$
$SIY = ||L \cdot S1||^2 / ||D \cdot Sd||^2 = |L|^2 / |D|^2$

Note the objective of the auxiliary path is to collect the replica of the interference signal S1, hence it is the desired signal for the path.

Then the SIR of the resultant output Z is

$SIZ = ||L^* \cdot Sd||^2 / ||D^* \cdot S1||^2 = |L|^2 / |D|^2$

An interesting fact is

$SIZ = SIY$

regardless of the main path signal SIX.

Since the main path antenna is usually larger than that of the auxiliary path with greater directivity, the above fact means the LMSE processing will rather degrade than improve the SIR performances of the system.

This adverse effect was first reported by Widrow, et.al [1] as a surprise in 1973.

The above adverse effect can be clearly understood by the signal space theory as follows.

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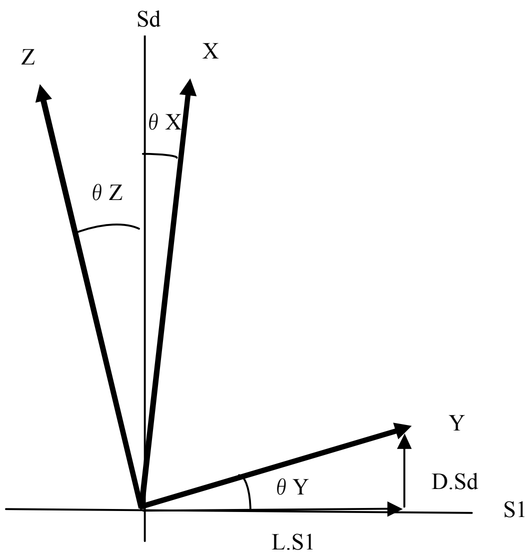

The signal space is defined by unit vectors Sd and S1 which are mutually orthogonal because they come from different origins.

Since Z must be orthogonal to Y, the angles in the above figure are equal $\theta Y = \theta Z$.

Those angles are related with the SIR of the signals by the formula;

$\tan^2(\theta Y) = 1 / SIY$

Since $\theta Y = \theta Z$, hence SIZ = SIY.

4. Angles between Signal Subspaces

As the output Z must be orthogonal to all auxiliary path signals {Yi} it must be orthogonal to the subspace spanned by those auxiliary paths signals.

We start with a simple 3-dimensional signal space.

$Y1 = D1 \cdot Sd + L11 \cdot S1 + L12 \cdot S2$

$Y2 = D2 \cdot Sd + L21 \cdot S1 + L22 \cdot S2$

The angles $\theta 1$, $\theta 2$ of Y1,Y2 against S1-S2 plane are

$\tan^2(\theta 1) = |D1|^2 \ / \ (|L11|^2 + |L12|^2)$

$\tan^2(\theta 2) = |D2|^2 \ / \ (|L21|^2 + |L22|^2)$

The subspace Y is spanned by Y1 and Y2 by a linear combination.

$Y = \alpha \cdot Y1 + \beta \cdot Y2$

where α and β are complex variables.

The angle θY of the Y plane against S1-S2 plane is defined as the maximum of the angle of the vector Y with variable coefficients α and β.

In the special case where L12 = L21 = 0 the following formula is derived by direct calculation.

$\tan^2(\theta Y) = \tan^2(\theta 1) + \tan^2(\theta 2)$

In the more general case L12 and L21 are non-zero, it can be shown also by direct calculation.

$\tan^2(\theta Y)$

$= (| D1. L21 - D2. L11 |^2 + | D1. L22 - D2.L12 |^2)$

$/ \ | L11.L22 - L12. L21|^2$

Note that it is absolute square sum of the solution of the equation.

[L].[x> = [D>

Let the solution <x] = (x1, x2), where <x] is the row vector associated with the column vector[x>.

Then

$\tan^2(\theta Y) = |x1|^2 + |x2|^2$

The formula can be interpreted as follows;

The signals Y1 and Y2 can be expressed in a vector and matrix form as,

[Y> = [D>.Sd + [L].[SI>

where <Y] = (Y1,Y2), <D]= (D1,D2) and

[L] = ([L1>,[L2>],

where <L1]= (L11, L21), and <L2]= (L12, L22)

Then the above formula is equivalent to the following orthogonalization of bases S1- S2 plane.

[L^][Y> = [L^].[D> Sd + [SI>

where [L^] is the inverse matrix of [L].

And <SI] = (S1, S2) are bases vectors of the SI subspace.

The above analysis can readily be generalized to multi dimensional cases where we have (m+1) original signals ; Sd, S1,S2,,,,Sm.

In more general cases where the above orthogonalization is not possible (ex. the number of original signals m is larger than the number of the auxiliary paths n or there is no inverse matrix for [L]) , the following theorem holds.

The subspace spanned by vectors Y1 and Y2 each with angles $\theta 1$ and $\theta 2$ against SI subspace (S1,S2 ,,,Sm)

toward base Sd
is separated from the SI subspace by the angle θ Y:

$$\tan^2(\theta Y) \geq \tan^2(\theta 1) + \tan^2(\theta 1)$$

the equality holds if the SI subspace components of Y1 and Y2 are mutually orthogonal.
It can be readily generalized to multi-dimensional cases.

The above formula can be also rewritten in terms of SIR;

$$SIZ = SIY \leq (SIY1^{-1} + SIY2^{-1})^{-1}$$

The above inequality tells the SIR of the LMSE output always degrades as the number of the auxiliary paths increases. If the number of the auxiliary paths exceeds the number of the original signals, then the Y- subspace reaches the whole signal space (Sd, S1, S2,-,,, Sm). Then the output Z must be trivially zero as it must be orthogonal to the whole signal space.

5. Methods to Improve LMSE Algorithm

The above analysis tells that the problem is caused by the leakage of the very desired signal Sd into the auxiliary paths. Considering this facts a number of methods were developed to improve the performance of LMSE algorithm.
An immediate solution is to get replicas of the interferences while there is no desired signal [7].
In echo cancellation system the replica of the interference signal is readily available because the interference is its own transmit signal reflected at the far end of the long haul transmission lines [6].

Decision Feedback Method
In digital communications the desired signal can be regenerated at the receiver with a good likelihood if the SIR is sufficiently high. Then the regenerated desired signal replica can be used to remove the desired signal component before the correlation measurement in order to eliminate the effect of the desired signal Sd. The method is called **decision-feedback**, has been widely used in digital communications [2][3][5].
We analyze a simple case where we have only the desired signal Sd and an interference signal S1.

$$X = Sd + I \cdot S1$$
$$Y = D \cdot Sd + L \cdot S1$$

Then the canceller output

$$Z = X - W \cdot Y$$
$$= (1 - W \cdot D)Sd + (I - W \cdot L) \cdot S1$$

From Z we regenerate a replica of the desired signal Sd' and subtract it from Z.
Let

$$Sd - Sd' = \varepsilon . Sd \qquad (|\varepsilon| \ll 1)$$

We apply an LMSE operation to Z by Sd';

$$Z' = Z - V \cdot Sd'$$

And

$$(Z', Sd') = 0,$$

Then we get

$$Z' = (1 - W \cdot D)\varepsilon Sd + (I - W \cdot L) \cdot S1$$

Now the correlation measurement is made between Z' and Y to control the coefficient W.

$$(Z', Y) = 0$$

which gives;

$$W = (\varepsilon D^* + L^* \cdot I) / (\varepsilon |D|^2 + |L|^2)$$
$$Z = A' \cdot (L^* \cdot Sd - \varepsilon D^* \cdot S1)$$

where $A' = (D \cdot I - L) / (\varepsilon |D|^2 + |L|^2)$
The SIR of the output Z is

$$SIZ = \| L^* \cdot Sd \|^2 / \| \varepsilon D^* \cdot S1 \|^2$$
$$= |L|^2 / |\varepsilon D|^2$$
$$= SIY / |\varepsilon|^2$$

If ε is small, then the resultant SIR is greatly improved.

The above analysis shows the regeneration of the desired signal and removal of it before the correlation measurement and control of the coefficient W is equivalent to reduction of the desired signal leakage into the auxiliary path signals.

$$Z' = (1 - W \cdot D)\varepsilon Sd + (I - W \cdot L) \cdot S1$$
$$Y = D \cdot Sd + L \cdot S1$$

Let

$$Y' = \varepsilon D \cdot Sd + L \cdot S1$$

Then

$$(Z', Y) = (Z, Y') \qquad (\varepsilon \text{ real})$$

Practically ε is close to BER (Bit Error Rate) which is very small in normal operations.

Hard Limiter Method
A replica of the desired signal can be also regenerated by a hard limiter if the SIR of the signal Z is sufficiently

high [8]. Then the above method can be applied to improve the performance of the LMSE operations.

The feature of the hard limiting method is its applicability to wide ranges of communications.

6. Applications to MIMO Communication Links

The multiple-input-multiple-output (MIMO) systems are normally processed in antenna levels. The LMSE method can realize a MIIMO system processed in IF or baseband by digital signal processing (DSP).

In a MIMO system the signals S1,S2,,,,Sm are all desired signals and we have the same number of receive paths;

$Yi = [j = 1, m] \Sigma Lij \cdot Sj$

$(i = 1,2,,,, m)$

For each Yi we do the following processing

$Zi = Yi - [j=1,m \text{ except for } i] \ \Sigma Wij \cdot Yj$

From Zi we regenerate a replica Si'of the desired signal Si and remove it from Zi to get Zi'. Then the correlation measurement is made between Zi'and Yj (excluding i) to control the coefficients Wij.

The resultant SIR of Zi will be

$SIZi = 1/| \varepsilon i \cdot \tan(\theta i')|^2$

Where θ i'is the angle of the subspace spanned by {Yj ; j = 1,2,,,, m except for i} against the basis subspace spanned by {Sj ; j = 1,2,,,, m except for i}.

The ε i is the relative error in regeneration of the signal Si; Si'- Si = ε i. Si.

7. Conclusion

The standard LMSE method that simply minimizes the power of the desired path subtracted by the auxiliary paths signals with adaptive weights can not achieve interference cancellation because the achieved SIR of the output is equal to or smaller than the inverse of the sum of the inverse SIRs of the auxiliary paths.

The problem can be clearly understood analyzing the function in the signal space spanned by the desired signal Sd and the interfering signals S1,S2,,,,Sm.

The element signals from different origins are un-correlated, or mutually orthogonal in the signal space.

A group of signals mutually orthogonal do not necessary mean they are the original signals. In fact the LMSE method achieves the output signal orthogonal to all auxiliary paths signals but its SIR is at most equal to that of the auxiliary paths which are in practice worse than that of the main path.

The signal space analysis of the LMSE method can clearly show the mechanism how the decision feed-back methods can improve the LMSE performances. The key factor is the regeneration of the original desired signal which is in essence the objective of the communication and elimination of it from LMSE measurement & control operations. Note a priori knowledge about the signal used for the communication plays the essential part in the improvement of the LMSE algorithm.

The improved LMSE method can be extensively applied to MIMO system implemented in IF or baseband utilizing DSP (Digital Signal Processing).

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