# Performances of Signalling System in Radio Communication Network 

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## Foreword

The signalling system forms an essential part of connection- oriented communication networks including PSTN (Public Switched Telephony Network) and various mobile communication networks. The mobile communication networks can be viewed as an extension of PSTN where the end terminals are connected through radio links. The radio links can be cellular zones of various sizes or satellites. A special requirement of the radio network in call connection control (CCC) is equipment of a random access request channel (RACH) and call announcement or paging channel (PCH).

In many mobile communication networks the RACH and PCH are of time division multiplex (TDM) channel composed of short time slots (TS) suitable to carry the small signalling messages. The user terminals (UT) are normally grouped to a number of paging groups (PG) to allow the UT an intermittent reception of the relevant PCH slots in low duty ratio; it can stay in sleep state most of the time, thus save the power consumption in idle states.

Another requirement is access grant channel (AGCH) to be used to assign a traffic channel (TCH) for user communications. A nother essential channel is broadcasting channel (BCCH) sent from the network control station (NCS) which disseminates such system parameters as system ID, cell or beam ID, the frequency and TDM definitions of RACH, PCH and AGCH. The PCH and AGCH can be combined as they are transmitted from the network through base stations (BS) toward UTs. The RACH is sent from UT to BS for initiation of mobile originated calls (MOC) and also such system control messages as registration, location updating or de- registration, etc.

The PCH and RACH are dedicated respectively for MTC (mobile terminated calls) and MOC (mobile originated calls) processing.. The AGCH is used for both MTC and MOC. Thus the AGCH combines both MTC and MOC processing into a common call control system.
The BCCH, PCH, AGCH, RACH are used commonly by all UTs and BS, hence are called common control channels (CCCH). This memorandum analyses the performances of call connection processing through CCCH.

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## 1. Structure of Mobile Communication Network



The radio communication is provided between the base stations (BS) and user terminals (UT) in each radio beam or cells. The communication link is maintained as the UT moves between different beams through hand over procedure coordinated among UT, BS, BSC and Mobile Switching Network.

Each base station (BS) is provided with a number of radio channels for system control and traffic services.

Each beam has a set of common control channels (CCCH) for common signalling control.

Forward- link CCCH
A broadcasting control channel (BCCH) is sent on frequency channel c0. The paging channel ( PCH ) and access- grant channel (AGCH) are sent on c1. The radio channel frequency for c 0 and c 1 are system parameters disseminated through BCCH.

An example CCCH structure in time division multiplex (TDM) mode is depicted in the folllowing figure.

Example with BCCH_TN = tn mod 25


Forward link CCCH parameters;
Number of Time Slots (TS) per BCCH frame ; 25
BCCH frame period ; 1/6(sec)
Number of PCH TS per BCCH frame ; [PCH_RES] (<11)

## Return link CCCH

RACH is a slotted ALOHA random access channel. The first RACH- window starts with the time slot of the BCCH for the beam and the window- length is 3 time- slots to secure reliable message transfer and allow the UTs sufficient timing error margin..

Number of RACH TS ; 50 (/sec)
For busy beams up to two frequencies may be assigned for the RACH.

## 2 Call set up procedure

## Mobile originated call (MOC)

(1) The UT sends a RACH message to the BS.
(2) The UT waits for a time equivalent to the minimum response time that is the round trip radio propagation delay and the processing time at the BS.
(3) After elapse of the response time the UT starts receiving all AGCH bursts to check if there is a response for the UT from the BS. If the response is detected, then the UT acquires the radio channels specified by the AGCH messages. Subsequent sequences follow on the assigned channel thus the AR\&G (Access Request and Grant) procedure is completed in success. The UT and BS then start a call connect procedure through the radio link to establish the call connection throughout the Mobile Switching Network. At completion of the call connection procedure, the end- to end user communication starts and continues until the users terminate the call. Then follows the call clearing procedure through the networks. After the call is cleared the radio channels are released and returned to the pool of idle channels managed by the BS.
(4) If there is no response for a fixed time period in step (2), the UT starts repetition trial if certain conditions are met, e.g. the number of repetitions has not reached the permitted maximum yet. If the conditions are not met then the UT completes the AR\&G procedure as a failure.
(5) The repetition of the AR\&G procedure starts with a randomisation process over a specified time period to decide the RACH timing to send the access request message. The procedure after sending the access message is the same as above.

Mobile terminated call (MTC)

## Idle state

The UT stays in idle mode and monitors the PCH for reception of possible call announcement. The grouping of UTs into different paging groups (PG) and time sharing mode scheduling of PCH allows the UT to stay in sleep state and intermittently wake up to monitor relevant PCH messages for possible call announcements.

Once the UT receives a PCH message addressed to it, it returns a response message to the BS through RACH. The subsequent procedure of MTC is similar to that of MOC.

## 3. Performance of the Access Request Processes

Let
$q(n)$; Average number of new access requests for the $n$ - th time window. $\mathrm{Q}(\mathrm{n})$; Average number of total RACH messages for the n - th time window. b; ratio ( $<1$ ) of re-transmitted RACH access request after a failure.

The probability that $k$ RACH messages are sent per a RACH time window (slots) ;

$$
P R[Q](k)=Q^{\wedge} k / k!/ e^{\wedge} \quad(k=0,1,2,3, \quad \ldots,) \quad(R A C H-1)
$$

Specifically the probability that no burst is sent is $\operatorname{PR}[Q](0)=1 / \mathrm{e}^{\wedge} \mathrm{Q}$.

Throughput $=$ the average number of successful messages transfer through a time window

- A single RACH burst is sent to the time window and the message is successfully regenerated at the BS.
- f; Probability of message loss due to transmission errors.

Then the throughput is;

$$
\begin{align*}
y & =(1-f) \cdot P R[Q](1) \\
& =(1-f) \cdot Q / e^{\wedge} Q \tag{RACH-2}
\end{align*}
$$

The average number of RACH messages for the $n$ - th time window $Q(n)$ must be the sum of the average number of the new access requests arising for the time window $q(n)$ and $a$ portion (b) of the RACH transactions failed in the previous AR\&G round.
Thus,

$$
\begin{equation*}
Q(n)=q(n)+b \cdot F(n-<R>) \cdot Q(n-<R>) \tag{RACH-3}
\end{equation*}
$$

Where $<\mathrm{R}>$ is the average- Repetition- period that is the number of time slots between successive transmission of access requests from a UT due to channel assignment failures. The $F(n)$ is the probability that a particular message transfer fails and expressed as;

$$
\begin{align*}
& F(n)=\{Q(n)-y(n)\} / Q(n) \\
&=1-(1-f) / e^{\wedge} Q(n) \tag{RACH-4}
\end{align*}
$$

The timing relationship of RACH repetitions is depicted in the following figure. Note that a particular access request can be repeated after [S, S+T] time slots. Inveresely a RACH messages for a particular Rach window can be repetitions from previous messages originally sent in the preceding time slots [S, S+T]. Considering the randomization is performed over T time slots, it is reasonable to assume the average repetition time interval; $\angle \mathrm{R}>=\mathrm{S}+\mathrm{T} / 2$.
$\angle R>$ is normally in the range of a second; short enough to assume that the stae variables, i.e. those probabilities $\mathrm{Q}, \mathrm{F}$, q remain constant in the time range of the average repetition; $\langle R\rangle$.


## RACH Timing for Repetition

The difference of the RACH traffic between time- slots separated by $<\mathrm{R}>$ is

$$
\begin{aligned}
D Q(n) \quad & =Q(n)-Q(n-<R>) \\
& =q(n)-\{1-b \cdot F(n-<R>)\} \cdot Q(n-<R>)
\end{aligned}
$$

$$
\begin{equation*}
\Leftrightarrow \quad q-(1-b) \cdot Q-b \cdot(1-f) \cdot Q \cdot e^{\wedge} Q \tag{RACH-5}
\end{equation*}
$$

We assume $Q(n)(=) Q(n-<R>)$,the change of the state is sufficiently slow.
((=) means nearly equal)
If $D Q>0, Q$ increases. If $D Q<0, Q$ decreases and if $D Q=0, Q$ remains the same, that is the state is in an equilibrium.

The state transition dynamics can then be clearly depicted by the phase space diagram as in the following.


Figure1-1 Phase space diagram of RACH

The figure shows that there are in general three stationary points Q1, Q2, Q3 of which Q1 and Q3 are stable and Q2 is unstable.

In the steady states where statistical equilibrium holds, the variables Q , F , and q meet the following relation;

$$
\begin{equation*}
Q \cdot(1-b \cdot F)=q \tag{RACH-6}
\end{equation*}
$$

or

$$
\begin{equation*}
q=(1-b) \cdot Q+b \cdot(1-f) \cdot Q \cdot e^{\wedge} Q \tag{RACH-7}
\end{equation*}
$$

and

$$
\begin{align*}
& F=1-(1-f) / e^{\wedge} Q  \tag{RACH-8a}\\
& y=(1-f) \cdot Q \cdot e^{\wedge} Q \tag{RACH-8b}
\end{align*}
$$

The probability of a successful RACH transfer in the $n$ - th attempt;

$$
\begin{equation*}
R(n)=(1-F) \cdot F^{\wedge} n \tag{RACH-9}
\end{equation*}
$$

Then the average number of re- attempts for the RACH transfer;

$$
\begin{align*}
<n> & =1 /(1-F) \\
& =e^{\wedge} Q /(1-f) \tag{RACH-10}
\end{align*}
$$

If the permitted number of re- attempts is limited to a finite number $N$, then

## Probability [RACH failure] $=\mathrm{F}^{\wedge} \mathrm{N}$ <br> Example

Let us assume the RACH traffic for a busy beam is 15.54 messages/sec (system design).
Since there are 50 RACH slots per second,

$$
\begin{equation*}
q=15.54 / 50=0.31 \tag{RACH-11}
\end{equation*}
$$

The transmission frame loss probability is assumed to be

$$
\begin{equation*}
f=0.05 \tag{RACH-12}
\end{equation*}
$$

If unlimited re- attempts are allowed then the above analysis can be applied as follows;

$$
\begin{equation*}
Q \cdot e-Q=q /(1-f)=0.326 \tag{RACH-13}
\end{equation*}
$$

Which gives two solutions;

$$
\begin{align*}
& \text { Q1 }=0.6, \\
& \text { Q2 }=1.6 \tag{RACH-14}
\end{align*}
$$

The second one is unstable. That is, if the RACH traffic Q happens to exceed Q2, the state becomes unstable. This state is to be avoided.
In the stable state, the failure rate F of an attempt and the average delay $<\mathrm{n}>$ are;

$$
\begin{align*}
& F=1-(1-f) / e Q=0.48 \\
& <n>=e^{\wedge} Q /(1-f)=1.91 \tag{RACH-15}
\end{align*}
$$

The average delay can be shortened If another channel is added for the RACH. Then;

$$
\begin{equation*}
q=15.54 / 100=0.16 \tag{RACH-16}
\end{equation*}
$$

For which the above equilibrium equation gives;

$$
\begin{equation*}
\mathrm{Q}=0.2 \tag{RACH-17}
\end{equation*}
$$

The average single failure rate and delays are;

$$
\begin{align*}
& F=1-(1-f) / e Q=0.22 \\
& <n>=e Q /(1-f)=1.28 \tag{RACH-18}
\end{align*}
$$

## 4 Performance of Paging Processes

### 4.1 Paging groups and PCH Time Slots

In order to save power of the UT during sleep mode, that is, when the terminal is not in use but occasionally wakes up to monitor the paging channel to check if it has a UT terminated call, most systems specify some paging group control. The UT sets are grouped into a number of $\mathrm{N} \cdot\left[\mathrm{PCH} \_\right.$RES] groups where both N and PCH $\_$RES are advised by BCCH.
[Paging Group Number for a UT]
$=[$ IMSI of the UT] $\bmod (\mathrm{N} \cdot[\mathrm{PCH} R E S])$
$=$ Remainder ([IMSI] / (N • [PCH_RES]) )
(PCH-1)
The paging capacity for a paging group of UT sets is;

$$
\begin{align*}
& \mathrm{C}[\mathrm{pg}] \quad=1 \text { per }[\text { Paging period }] \\
&=3(\text { calls } / \mathrm{sec} \cdot \text { For } \mathrm{N}=2) \\
&=0.5(\text { call } / \mathrm{sec} \text { for } \mathrm{N}=12) \tag{PCH-2}
\end{align*}
$$

The total paging capacity is given by;

$$
\begin{align*}
C[P] \quad= & 6 \times[P C H R E S] \\
& =<60 \quad(\text { calls } / \mathrm{sec}) \tag{PCH-3}
\end{align*}
$$

### 4.2 Queueing Performance of PCH

The paging process can fail due to the following causes;
(1) PCH message loss due to transmission error
(2) Target UT is not monitoring the particular PCH;

Successful case;
A dedicated channel is assigned through successful Paging, Access Request, Access Grant (PAR\&G) procedure by the MSC. Only after both the UT and the BS begin transactions by the assigned channel (TCH) the success of the paging is confirmed by the MSC.

Failure case;
If the success is not detected before the elapse of the timer, the MSC sends another PAGING request signal indicating a repetition.

The probability of a paging failure E is given by;

$$
\begin{align*}
\mathrm{E} & =\mathrm{E}^{\prime}+\mathrm{B}-\mathrm{E}^{\prime} \cdot \mathrm{B} \quad \text { (for initial attempt through one beam) } \\
& =\mathrm{E}^{\prime} \quad \text { (if all possible PCHs are used for paging) } \tag{PCH-4}
\end{align*}
$$

Where $B$ is the probability the target UT is not monitoring the PCH.
where $E^{\prime}$ is the aggregate failure probability of paging failure occurring in PCH, RACH on AGCH transfers.
In the following we assume that the UT is monitoring the correct PCH hence, E=E' .

The probability of a successful PCH message transfer in the $n$ - th attempt is;

$$
P(n)=(1-E) \cdot E N(n-1) \quad(n=,<M p) \quad(P C H-5)
$$

If the Mp-th attempt fails, then the paging is deemed as failed. The probability of failure is obtained by adding all $P(n)$ for $n=M p, M p+1, M p+2, \quad$, ,.

$$
\begin{align*}
\operatorname{Pr}[\text { paging failure }] & =P(M p+1)+P(M p+2)+, \ldots, \\
& =E^{\wedge} M p \tag{PCH-6}
\end{align*}
$$

In another word,

$$
\begin{equation*}
\operatorname{Pr}[\text { Paging success }]=1-\mathrm{EMp} \tag{PCH-7}
\end{equation*}
$$

The average number of paging repetition for successful cases is;

$$
\begin{aligned}
& <n[\text { success }]>=P(1)+2 P(2)+3 P(3)+, \ldots+M p \cdot P(M p) \\
& \quad=1 /(1-E) \cdot\left\{1-\quad(M p+1) \cdot E^{\wedge} M p+M p \cdot E N(M p+1)\right\} \quad(P C H-8)
\end{aligned}
$$

Note that in the limit Mp àinfinity, $\langle n>$ à $1 /(1-E)$.

The average number of message transmissions for failure cases is

$$
\begin{equation*}
<n[\text { failure }]>=M p \cdot E^{\wedge} M p \tag{PCH-9}
\end{equation*}
$$

The average number of message transmissions including both successful and failure cases is

$$
\begin{align*}
<n> & =<n[\text { success }]>+<n[\text { failure }]> \\
& =\left(1-E^{\wedge} M p\right) /(1-E) \\
& =1+E+E^{\wedge} 2+,,+E^{\wedge}(M p-1) \tag{PCH-10}
\end{align*}
$$

Note for Mp à infinity, then <n> à $1 /(1-E)$

Thus the traffic on PCH increases by <n> times the net paging requests due to failures in PAR\&G procedure.

Stability condition;
The rate a page moves out of the queue is $1 /<n>$, where $<n>$ is the average number of page repeat. On the other hand the number of input pages into the queue is $p$ for every paging period.

The average increase of the length of the queue;

$$
\begin{equation*}
\mathrm{d}<\mathrm{L}>=\mathrm{p}-1 /<\mathrm{n}> \tag{PCH-11}
\end{equation*}
$$

The condition that prevents the infinite growth of the queue length is;

$$
\begin{align*}
& p<1 /<n>\text {, or } \\
& p \cdot<n><1 \tag{PCH-12}
\end{align*}
$$

If $\langle\mathrm{n}>$ is 1.1, then $p$ must be smaller than 0.91.

The average time required for paging when the length of the queue is $L$ is
$<$ Paging time $>=(1+<L>) \cdot<n>\cdot$ [Paging period]
(PCH-13)
The paging period is $N$ CCCH frames ranging from $1 / 3(\mathrm{~N}=2)$ to $2(\mathrm{~N}=12)$ seconds.
The paging time directly affects the mobile- terminated call set up time.

If a new call arises while there are previous calls waiting for paging, then it is put into the queue until the previous pages are cleared.
Let $p$ be the average number of call generations per paging slot. Then the probability $P(k)$ that there arise a number $k$ of calls generated during the period is given by the following Poisson formula.

$$
\begin{equation*}
P(k)=p^{\wedge} k / k!\cdot e^{\wedge} p \tag{PCH-14}
\end{equation*}
$$

Let $S[m]$ be the probability of the state that there are $m$ pages in the PCH queue at the beginning of a time slot and $\mathrm{S}[\mathrm{m}]$ ' at the beginning of the next paging slot. Then the state transition equation is given as follows;

$$
\begin{align*}
S[m]^{\prime}=(1-E) & \cdot P(0) \cdot S[m+1] \\
& +\{(1-E) \cdot P(1)+E \cdot P(0)\} \cdot S[m] \\
& +\{(1-E) \cdot P(2)+E \cdot P(1)\} \cdot S[m-1]+, \ldots, \prime \\
& ,+\{(1-E) \cdot P(m)+E \cdot P(m-1)\} \cdot S(1) \\
& +P(m) \cdot S[0] \tag{PCH-15}
\end{align*}
$$

In the steady state under consideration; $\mathrm{S}^{\prime}=\mathrm{S}$.
We assume the solution in the form of

$$
\begin{equation*}
\mathrm{S}[\mathrm{~m}]=(1-\mathrm{x}) \cdot \mathrm{xm} \tag{PCH-16}
\end{equation*}
$$

We also assume that the length of the queuing memory is infinite for simplicity. This assumption is practical if x is small.

Let

$$
\begin{equation*}
y=p / x \tag{PCH-17}
\end{equation*}
$$

Then, it can be shown that the above characteristic equation is quite simplified as follows;

$$
\begin{equation*}
e^{\wedge} p / p \cdot y=(1-E+E / p) \cdot e^{\wedge} y \tag{PCH-18}
\end{equation*}
$$

or

$$
\begin{equation*}
e^{\wedge} y / y=e^{\wedge} p / p /\{1+E \cdot(1 / p-1)\} \tag{PCH-19}
\end{equation*}
$$

Numerical example;

Suppose the paging period is set to 2 BCCH frames or $1 / 3(\mathrm{sec})$ and ten time- slots are reserved for paging in a BCCH frame. Then the PCH capacity is $\mathrm{Cpg}=60$ (messages/sec)

We try to find out the PCH performances when net $\mathrm{P}=3.6$ (calls/sec) is applied to the system and the worst case average reprtition is 3 (times) hence the peak PCH traffic is $3 \times 3.6=10.8$ (PCH messages per second)
(System design).
The average telephony call duration is in the range of $D=150$ (sec).
Then the traffic for the system $=P * D=3.6 \times 150=540$ (Erlang).

Then

$$
\begin{equation*}
p=10.8 / 60 \quad=0.18(\text { per }[\text { Paging period }]) \tag{PCH-20}
\end{equation*}
$$

The failure probability for PCH radio transmission loss;

$$
\begin{equation*}
\mathrm{e}=0.02 \text { (System design) } \tag{PCH-21}
\end{equation*}
$$

Overall PCH- RACH- AGCH message loss rate;

$$
\begin{equation*}
E^{\prime}=0.1 \tag{PCH-22}
\end{equation*}
$$

We also assume that the maximum repetition times is,

$$
\begin{equation*}
\mathrm{Mp}=3 \tag{PCH-23}
\end{equation*}
$$

Then the probability of resultant paging failure is from equation (PCH-6),

$$
\begin{array}{rlrl}
\operatorname{Pr}[\text { paging failure }] & =E^{\wedge} \mathrm{Mp} & & (\mathrm{PCH}-24)  \tag{PCH-24}\\
& =10-3 & (\mathrm{PCH}-25)
\end{array}
$$

The average number of paging repetition is calculated by equation ( $\mathrm{PCH}-8$ );

$$
<n>=1 /(1-E) \cdot\left\{1-\quad(M p+1) \cdot E^{\wedge} M p+\cdot E^{\wedge}(M p+1)\right\}
$$

(=) 1.11
(PCH-26)
The queuing performances can be now calculated by using the characteristic root given in the previous section. We analyse the case $\mathrm{E}=0.1$. The result is given in the following table.

| PCH queuing performance for PCH message loss probability $\mathrm{E}=0.1$ |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| p $\mathrm{e}^{\mathrm{y}} / \mathrm{y}$ y x $\mathrm{S}[0]$ $\mathrm{S}[1]$ $<\mathrm{m}>$ <br> 0.01 9.267 3.5 0.00286 0.997 0.00285 0.00287 <br> 0.1 6.907 3.0 0.0333 0.977 0.0325 0.0341 <br> 0.18 4.570 2.4 0.0750 0.925 0.0694 0.0811 <br> 0.5 3.000 1.8 0.280 0.720 0.202 0.357 <br> 0.9 2.703 1.1 0.818 0.182 0.149 4.500 |  |  |  |  |  |  |

## 5. Overall Perfomances of Call Connction Control system

In the following analysis a satellite link is assumed but the same model is applicable to any mobile radio communication networks. The feature of satellite system is the great size of the sevice area hence the CCC system must handle a large traffic capacity.

### 5.1 Model of CCC system

Both mobile- originated call (MOC) and mobile-terminated call (MTC) share the same access request and grant (AR\&G) procedure. In addition MTC has paging procedure to trigger the process.
p; Paging traffic (Average number of Paging requests per PCH Time- slot) q; Access request traffic
(Average number of RACH messages per RACH window (Time Slot))
E; Failure probability of an MTC attempt
F; Failure probability of an MOC attempt


## Model of CCC system

The repetition of the MOC and MTC blocks can be simplified applying the feedback transfer function formulae. The simplified model is depicted in the following figure.


Traffic model of the satellite link signalling system

### 5.2 Failure Probability of MOC and MTC Attempts

From the above traffic model the following relations stand.

$$
\begin{aligned}
& p^{\prime}=p /(1-E) \\
& q^{\prime}=q /(1-F) \\
& Q^{\prime}=q^{\prime}+(1-P) \cdot p^{\prime}
\end{aligned}
$$

Where $P$ is the probability of PCH message loss due to transmission errors (Frame Error ate, FER ).

The throughputs for MOC and MTC are;

$$
\begin{aligned}
& y=q^{\prime} \cdot(1-h) \cdot(1-f) \cdot e^{\wedge} Q \\
& x=p^{\prime} \cdot(1-g) \cdot(1-e) \cdot e^{\wedge} Q \cdot(1-P)
\end{aligned}
$$

Where,
e ; Failure probability for RACH transfer for MTC
f ; Failure probability for RACH transfer for MOC
g ; Failure probability for AGCH transfer for MTC
h ; Failure probability for AGCH transfer for MOC
Q; Combined RACH traffic
The meaning of above equations is clear if it is noted that $e^{\wedge} Q$ gives the probability that there will be no collision in the RACH.

The single AR\&G failure probabilities for MTC and MOC are;

$$
\begin{aligned}
E^{\prime} & =\left(p^{\prime}-x\right) / p^{\prime} \\
& =1-(1-P) \cdot(1-e) \cdot(1-g) \cdot e^{\wedge} Q
\end{aligned}
$$

$$
\begin{aligned}
F^{\prime}= & \left(q^{\prime}-y\right) / q^{\prime} \\
& =1-(1-f) \cdot(1-h) \cdot e-Q
\end{aligned}
$$

The single MTC channel assignment failure $E^{\prime}$ is ;

$$
\begin{aligned}
E^{\prime}= & 1-(1-P) \cdot\left(1-E^{\prime}, N\right) \\
& =P+(1-P) \cdot E^{\prime} \cdot N
\end{aligned}
$$

Where $N$ is the number of repetitions permitted for the AR\&G procedure.
Then the AR\&G failure probability for the MOC is

$$
F=F^{\prime} N
$$

If up to M repetitions are permitted for MTC, then the final failure probability for the MTC is expressed as;

$$
\mathrm{E}=\mathrm{E}^{\prime} \mathrm{M}
$$

### 5.3 Combined MOC and MTC Access Request and Grant (AR\&G) Performances

We assume up to $N$ trials for the AR\&G procedure is permitted. In addition, up to M repetitions are permitted for the MTC.
Then the AR\&G traffic from the MOC is;

$$
\begin{gathered}
q \cdot 1 /(1-F) \cdot\left(1+F^{\prime}+F^{\prime} 2+, \ldots+F^{\prime}(N-1)\right) \\
=q /\left(1-F^{\prime}\right)
\end{gathered}
$$

where the first factor $(q)$ is the original traffic, the second factor $1 /(1-F)$ is the rate of increase due to the repetition of call set- up trial by the UT user and the third factor is the rate of increase of the traffic due to the repetition of RACH and AGCH procedure.

In the same manner the traffic for the AR\&G process from MTC is;

$$
\begin{aligned}
& p \cdot 1 /\left(1-E^{\prime} M\right) \cdot\left(1+E^{\prime}+E^{\prime} 2+,,+E^{\prime}(M-1) \quad\right) \cdot(1-P) \\
& \quad\left(1+E^{\prime}, \quad+E^{\prime}, 2+,,+E^{\prime}(N-1)\right) \\
& =p /\left(1-E^{\prime}\right)
\end{aligned}
$$

The fourth factor in the above equation (1-P) is the success rate of PCH message transfer. Naturally only successful pages trigger the AR\&G process.

The input traffic Q for the AR\&G process is therefore;

$$
Q=q /\left(1-F^{\prime}\right)+p /\left(1-E^{\prime}\right)
$$

From previous equations the above equation can be modified as follows;
$\mathrm{Q} \cdot \mathrm{e}^{\wedge} \mathrm{Q}=\mathrm{q} /(1-\mathrm{h}) /(1-\mathrm{f})+\mathrm{p} /(1-\mathrm{g}) /(1-\mathrm{e})$

The meaning of the above equation is clear. The AR\&G process is the random access system for the input traffic, which is increased from the original traffic to compensate for the transmission losses. The left hand side is the throughput of the RACH, hence the equation states the equilibrium of the input and output signalling traffic.

### 5.4 Numerical design of a mobile satellite system

### 5.4.1 Traffic of the system

MTC
MT Calls set- up
11.4 (calls per second)

Call announcements(Paging)
36 (PCH messages per sec)
MOC
user communications
Normal calls 11.4 (calls per second)
Emergency calls 6
MOC system control communications
UT registration
16.3

UT de-registration 16.3
UT location update 24.6

## Total traffic

From the above table the traffic for a satellite in final stages is 11.4 calls/sec for MTC and 74.6 calls/sec for MOC of which 17.4 calls/sec are for user communications and the rest system control communications. It is expected that about $10 \%$ of the traffic will concentrate in a busy beam. In most beams the number of RACH windows per second is 50.

Based on the above assumptions, the RACH traffic per Time- slot in a busy beam is;

$$
\begin{aligned}
& p=1.14 / 50=0.0228 \\
& q[\text { user }]=1.74 / 50=0.0348 \\
& q[\text { control }]=5.72 / 50=0.1144
\end{aligned}
$$

## Traffic in Erlang;

The total traffic for the satellite is 22.8 user- calls/sec.

If we assume the average call duration is 150 seconds, the traffic will be $22.8 \times 150=3420$ Erlang.

### 5.4.2 Transmission error probabilities

We assume that the transmission errors for MOC user communications are negligible because users will first secure a clear view of the satellite before initiating a call. For the other MOC system communications as Location Updating, registration, IMSI detach, etc. the transmission conditions are assumed to be similar to that for the MTC, namely in poor conditions in many cases.

Thus the following values are assumed;
MTC .

$$
\begin{array}{cl}
\text { Paging error probability } & ; \mathrm{P}=0.1 \\
\text { Access request failure probability } & ; \mathrm{e}=0.1 \\
\text { AGCH failure probability } & ; g=0.2
\end{array}
$$

(because of 2 bursts length)
MOC user communications
Access request failure probability ;f(=) 0
AGCH failure probability $\quad ; \mathrm{h}(=) 0$
MOC system controls
Access request failure probability; $f=0.1$
AGCH failure probability $; h=0.2$
5.4.3 The traffic through the combined RACH

Then the equilibrium equation of the equivalent RACH is given as follows;

$$
\begin{aligned}
\mathrm{Q} \cdot \mathrm{e}^{\wedge} \mathrm{Q}=\mathrm{q}[\text { user }] \quad & +\mathrm{q}[\text { control }] /(1-\mathrm{f}) /(1-\mathrm{h}) \\
& +\mathrm{p} /(1-\mathrm{e}) /(1-\mathrm{g}) \\
= & \mathrm{q}[\text { user }]+1.39 \mathrm{q}[\text { control }]+1.39 \mathrm{p} \\
= & 0.2255 \text { (busy beam })
\end{aligned}
$$

Above equilibrium equations give the steady state RACH traffic

$$
\text { Q } \quad=0.306 \quad \text { (final stage, busy beam) }
$$

Then the single AR\&G failure probabilities for MTC and MOC are;

$$
\begin{aligned}
& E^{\prime}= 1-(1-P) \cdot(1-e) \cdot(1-g) \cdot e-Q \\
&=0.523 \text { (final stage) } \\
& F^{\prime}=1-(1-f) \cdot(1-h) \cdot e-Q
\end{aligned}
$$

$$
\begin{aligned}
& =0.264 \text { (for user) } \\
& =0.470 \text { (for control) }
\end{aligned}
$$

The single MTC channel assignment failure $E^{\prime}$ is ;

$$
E^{\prime}=P+(1-P) \cdot E^{\prime} \quad N=0.229
$$

Where $\mathrm{N}=3$ is assumed for the number of repetitions permitted for the AR\&G procedure. Then various signalling performances can be estimated based on the above parameters.

### 5.4.4 MOC performances

Failure rates;

$$
\begin{aligned}
\mathrm{F}= & \mathrm{F}^{\prime} & N & \\
& =0.0184 & & \text { (for user) } \\
& =0.104 & & \text { (for control) }
\end{aligned}
$$

Average number of total repetitions;

$$
\begin{aligned}
&<n>=\left(1-F^{\prime} N\right) /\left(1-F^{\prime}\right) \\
&=1.33 \text { (for user) } \\
&=1.70 \text { (for control) }
\end{aligned}
$$

### 5.4.5 MTC performances

Failure rate;

$$
\begin{aligned}
E=E^{\prime} M & \\
& =0.0524 \quad \text { (busy beam) }
\end{aligned}
$$

Where we assume up to $M=2$ paging trials in total.
The average number of total message transmissions is;

$$
\begin{aligned}
<n>= & \left(1-E^{\prime} M\right) /\left(1-E^{\prime}\right) \\
& =1.229 \quad \text { (final stage, busy beam) }
\end{aligned}
$$

The average number of paging repetition for successful cases is;

$$
\begin{aligned}
<n[\text { success }]>=<n>-M & \text { E' } M \\
=1.120 & \text { (busy beam) }
\end{aligned}
$$

Average length of Paging queue; $\mathrm{S}[\mathrm{m}]=(1-\mathrm{x}) \cdot \mathrm{x}^{\wedge} \mathrm{m}$

|  | p | x | $\mathrm{S}[0]$ <br> $=1-\mathrm{x}$ | $\mathrm{S}[1]$ <br> $=\mathrm{x} \cdot(1-\mathrm{x})$ | $\mathrm{S}[2]$ <br> $=(1-\mathrm{x}) \cdot \mathrm{x}^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Final stage, <br> busy beam | 0.0228 | 0.0671 | 0.933 | 0.0626 | 0.0042 |

- End-

