Generalized Trans Multiplexer

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Foreword

The trans-multiplexer (TMUX) is an example to show the power of Digital Signal Processing (DSP) in communication technology. The TMUX was developed and widely used for the TDM-FDM converter at switches in telephony networks. The switch is operated in TDM modes and the long distance transmission lines were operated in FDM modes hence Time-Frequency conversion was required at the input and output interfaces of the switches.

Recently OFDM which is a form of TMUX has found extensive applications in digital broadcasting, ADSL, mobile communication and wireless LAN systems.

The above applications have utilized the simplicity feature of the TMUX. In other applications which require flexibility in bandwidth management, those TMUX will not be applicable because of the fixed channel spacing in TMUX and bandwidth overlapping in OFDM.

This memorandum describes a generalized TMUX which allows channel bandwidth greater than the channel spacing thus can realize FDM networks capable to carry signals of variable bandwidths..

Contents

1. Z transform representation of sampled sequences

In DSP a time signal x(t) is sampled at time intervals T for signal processing.
The sampled sequences are expressed as follows;

x[T](t) = [n=0,∞]∑ x(nT). δ(t-nT)

where δ(t) is Dirac delta function, which expresses the time instant at t=0.
We apply the Laplace transform for the above sampled sequence;

X[T](s) = [n=0,∞]∑ x(nT).e^(-nT.s)

By setting

z = e^(sT)

X[T](z) = [n=0,∞]∑ x(nT). z^(-n)

If we set x(nT) = x(n), then we obtain a general expression with T omitted;
Let X[T](z)=X(z), then;

X(z) = [n=0,∞]∑ x(n). z^(-n)

Thus the Z transform is expressed independent of the sampling period T. However, the sampling period is important because of the sampling theorem.

Frequency spectra of sampled sequences

The expression of the sampled sequences can be modified as follows;

x[T](t) = [n=0,∞]∑ x(nT). δ(t-nT)
= x(t) . [n=-∞,∞]∑ δ(t-nT)
= x(t). 1/T.[n=-∞,∞]∑ e^(j(2π/T).n.t) (j^2 = -1)

Note the last part is derived by the Fourier series representation of the impulses sequences.
Then the Fourier transform of the sampled sequences is given as follows;

X[T](ω) = 1/T.[n=-∞,∞]∑ X(j{ω+n.2π/T) (j^2 = -1)

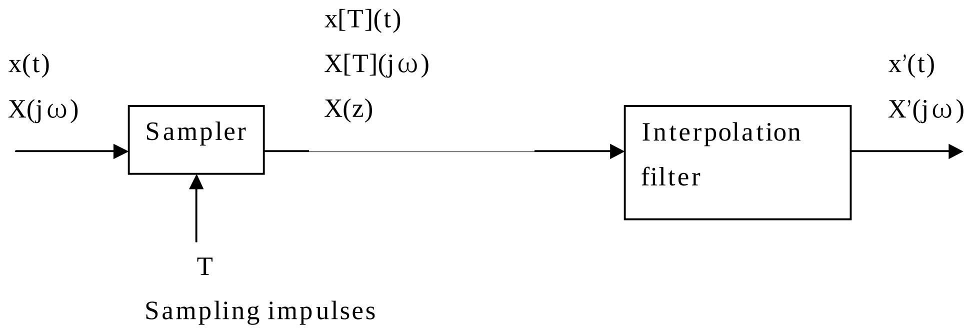
Where X(jω) is the frequency spectra of x(t);

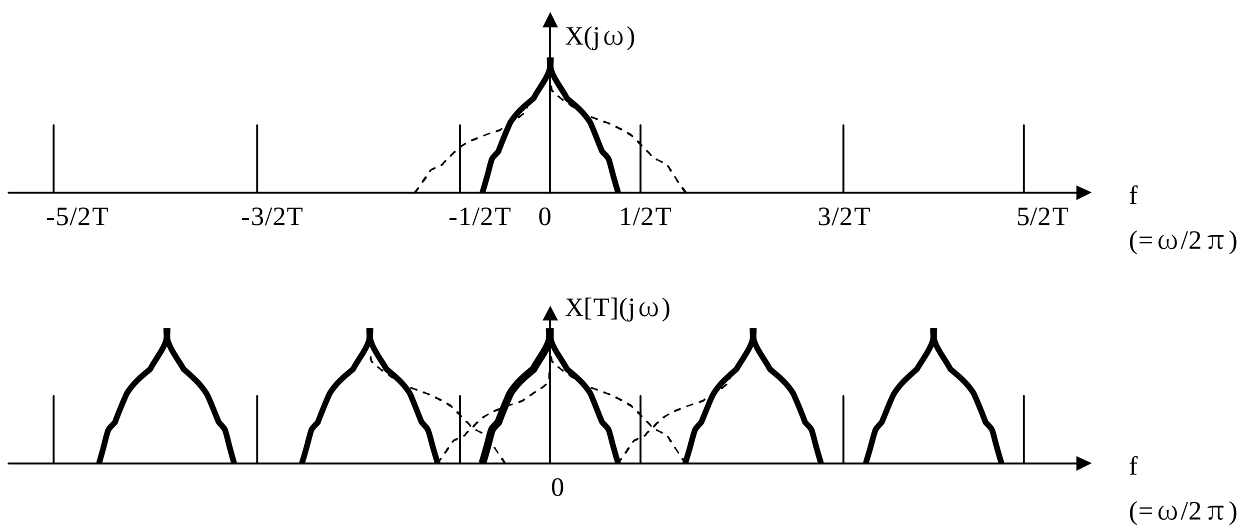
X(jω) = [t =-∞,∞]∫ x(t) .e^(-jωt) dt

Sampling theorem

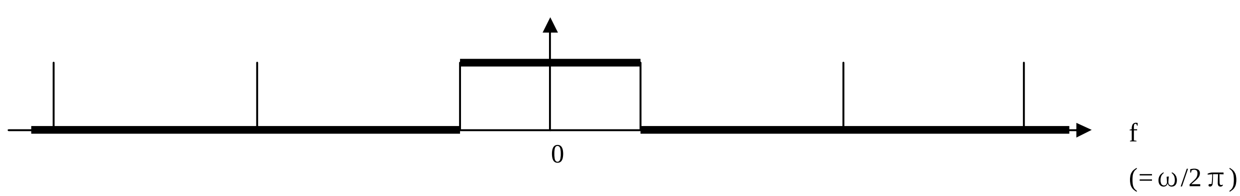
The bandwidth of the signal must be limited to [-1/2T, 1/2T] in order to avoid deformation of the signal caused by the sampling with period T. Then the original signal can be faithfully recovered from the sampled sequences by putting it through an ideal frequency filter of constant amplitude limited to [-1/2T, 1/2T].

The meaning of the sampling theorem will be clear from the following figure.

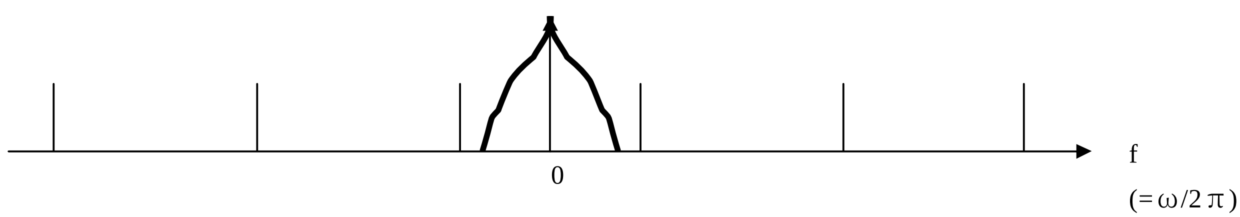




Ideal Interpolation filter



Interpolation output

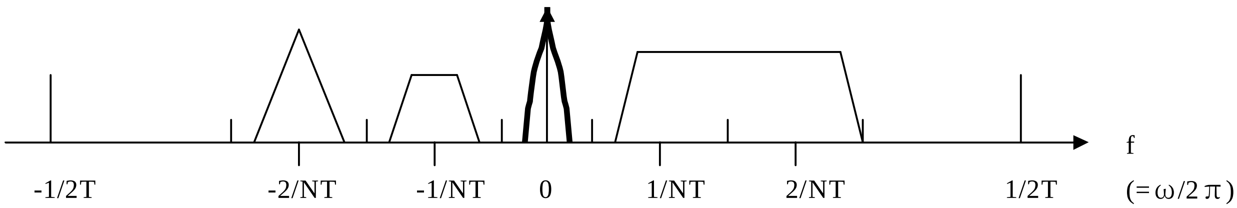


2. Generalized TMUX multiplexer

We will multiplex K signals over N frequency channels with the channel spacing

$$\triangle f = 1/(NT)$$

The total frequency bandwidth is limited to [-1/2T, 1/2T].



Our aim is to design a generalized TMUX which allows multiplexing of various types of modulated signals of various bandwidths.

[1] Sub-filter decomposition of digital filter

Let the z-transform of a filter

G(z) = [l=0,L-1]∑ g(l). z^(-l)

This is modified as follows

G(z) = [i=0,N-1]∑ z^(-i).G[i](z^N)

Where G[i](z^N) is called the i-th sub-filter and defined by

G[i](z^N) = [l=0,L/N-1]∑ g(l.N+i).z^-N

[2] Channel filtering and frequency conversion to FDM channels

An input signal is expressed by the following z transform;

X(z) = [n]∑ x(n). z^(-n)

We first put this into the filter G((z) to get

G(z).X(z) = [i=0,N-1]∑ z^(-i).G[i](z^N) . X(z)

Then we convert the frequency position of the signal from 0 (Hz) to k.△f = k/(NT) by the following variable conversion

z→ e^(- j2π/N .k) z (note z =. e^(j2π.f.T) and let f→ f – k/NT)

Let us denote the output by Y(z;k)

Y(z;k) = [i=0,N-1]∑ z^(-i).W^(-k.i).G[i](z^N) . X(z;k)

Where

W = e^(- j2π/N)

X(z;k) = [n]∑ e^(j2πk/N.n). x(n). z^(-n)

[3] Frequency Multiplexing

The Y(z;k) are multiplexed to give the multiplexer output

$$Y(z) = [k=0,N-1]\sum Y(z;k)$$
$$= [k,i=0, N-1]\sum\sum z\hat{}(-i).W\hat{}(-k.i).G[i](z\hat{}N) . X(z;k)$$
$$= [i=0, N-1]\sum z\hat{}(-i).G[i](z\hat{}N). \underline{[k=0, N-1]\sum W\hat{}(-k.i). X(z;k)}$$

The underlined part is the summation over k.

We can choose N as power of 2 and apply FFT to the summation over k.

[4] Structure of generalized TMUX Multiplexer

The above results are structured to the following circuit.

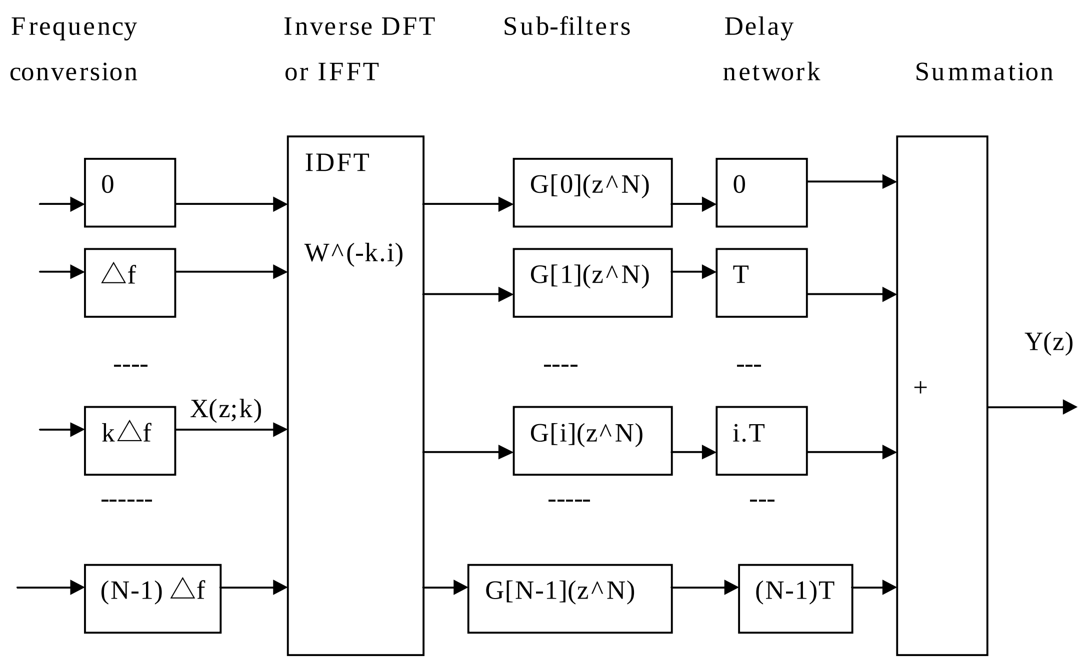


Fig. 2-1 Structure of Generalized TMUX multiplexer

The generalized TMUX operates at the speed of 1/T Hz. Thus the signal bandwidth can be expanded as much as [-1/2T, 1/2T]. It should be noted that all signals share the same filter G(z). The frequency characteristics are determined by G(z). The generalized TMUX can allow the bandwidth of each channel greater than the spacing frequency Δf, thus gives a flexibility in design of the communication network.

3. Generalized TMUX de-multiplexer

[1] Sub-filter decomposition of digital filter

We select the receive filter matched to the transmit filter G(z). The matched filter has the impulse response reversed in time domain. Hence the z-transform of a receiver filter shall be

H(z) = z^(-(L-1)) [l=0,L-1]∑ h(l). z^(l)

h(l) = g(l)

This is modified as follows

H(z) = z^(-(L-1)).[i=0,N-1]∑ z^(i).H[i](z^N)

Where H[i](z^N) is called the i-th sub-filter and defined by

H[i](z^N) = [l=0,L/N-1]∑ h(l.N+i).z^N

[2] Frequency conversion of the receive filter at k.Δf

In order to select the signal at frequency k.Δf we need a filter tuned at f = k.Δf.

By variable conversion

z→ e^(-j2π/N .k) z = W^k .z (z =. e^(j2π.f.T) and let f→ f - k/NT)

Then

H(z;k) = z^(-(L-1)).[i=0,N-1]∑W^(k.i). z^(i).H[i](z^N)

[3] Frequency de-multiplexing

Then the receive signal R(z) is put through H(z;k) to select the signal existing at the k-th channel Y(z;k).

Y(z;k) = H(z;k).R(z) = z^(-(L-1)). [i=0,N-1]∑W^(ki).H[i](z^N).z^i.R(z)

[4] frequency conversion to 0(Hz)

The above signal is centered at f = k.Δf.. Therefore it needs to be frequency converted to obtain the signal at base band.

Let

Y(z;k) = [n]∑ y(n). z^(-n)

Then

Y(z) = Y(z;k;-k) = [n]∑e^(-j2πk/N.n). y(n). z^(-n)

The circuit structure of the generalized TMUX is given in the following figure.

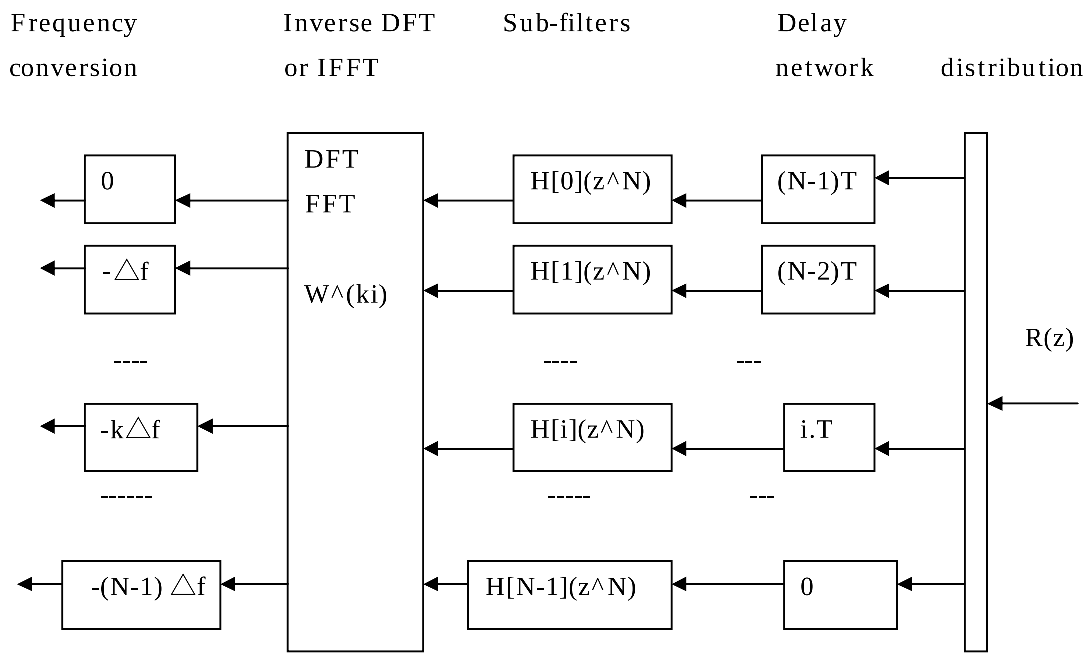


Fig. 3-1 Structure of Generalized TMUX de-multiplexer

Note the reverse structure of the de-multiplexer to the multiplexer.

A brief summary of The DFT (Discrete Fourier Transform) is given here.
For W = e^(-j2π/N),

1 + W^k + (W^k)^2 +(W^k)^3 +,,,,+ (W^k)^(N-1)   = N (k=0)
                                                                                  = 0 (otherwise)

For N samples; x(0),x(1), x(2),,,,,,x(N-1), the DFT X(k) is defined as;

X(k) = [n=0,N-1]∑ x(n).W^(kn)

The inverse DFT is given by

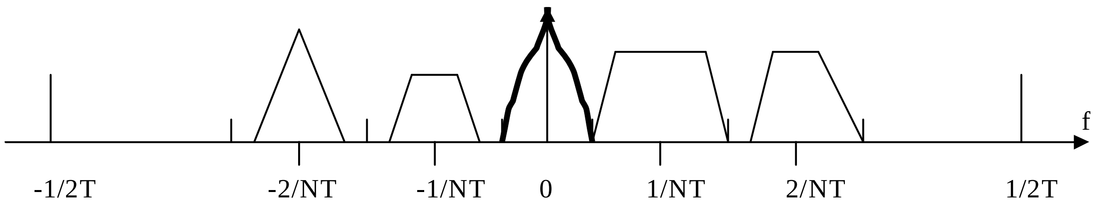
x(m) = 1/N. [k=0,N-1]∑ X(k).W^(-km)

The inverse formula is derived straight forward from the above formula;

The distributor and the delay circuit in the figure can be simply implemented by a shift register.

4. Conventional TMUX

Historically the trans-multiplexer was developed for telephony networks where all signals were of the same bandwidth. The voice signals were bandwidth limited to 3.4(kHz, 3dB) and frequency multiplexed with Δf = 4(kHz) spacing.
The frequency spectrum of the conventional TMUX is depicted in the following figure.



The channel uniformity can simplify the structure of the trans-multiplexer.
(1) No frequency conversion processing at the base band
Each traffic signal is sampled at the rate of Δf = 1/(NT). Then the original signals have all harmonics at k.Δf, hence no frequency conversion at base band is necessary.
(2) The delay networks and summation/distribution circuits
can be simplified as the multiplexed signals operate at the rate of 1/T (Hz) exactly N times the rate of the N branches circuits. This enables the delay networks and the summation and distribution circuits implemented by shift-registers with parallel data set functions widely used in P/S and S/P (P : parallel, S : serial) converters.
The structure of the conventional TMUX is depicted in the following.

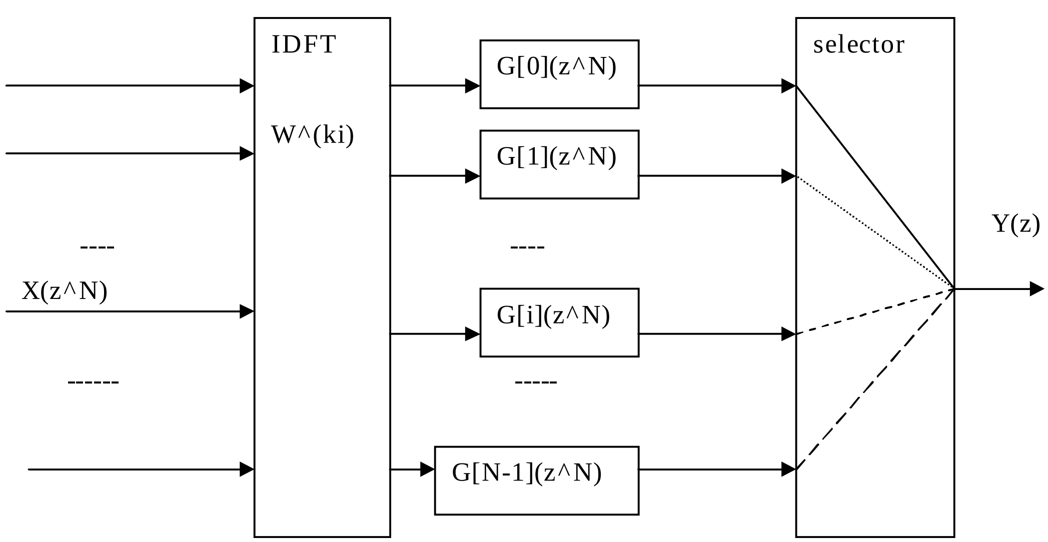


Fig. 4-1 Structure of conventional TMUX multiplexer

The output of the de-multiplexer needs only signals at the rate of 1/(NT). This can be utilized for N times reduction of the receiver processing. The down sampling can be made the front end of the receiver as shown in the following figure.

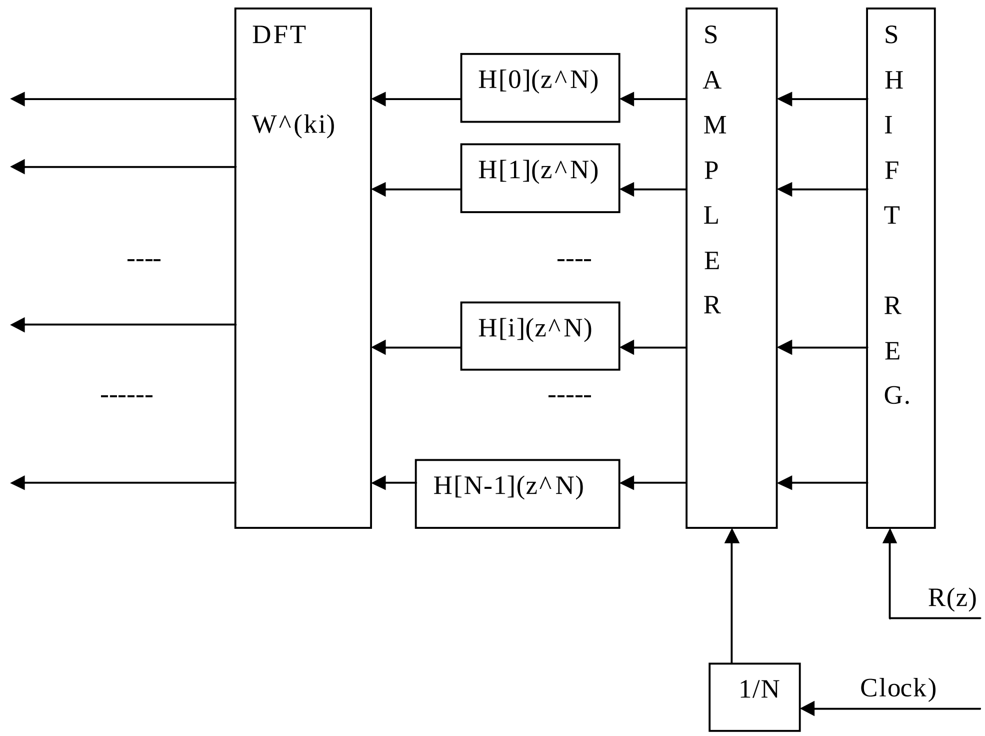


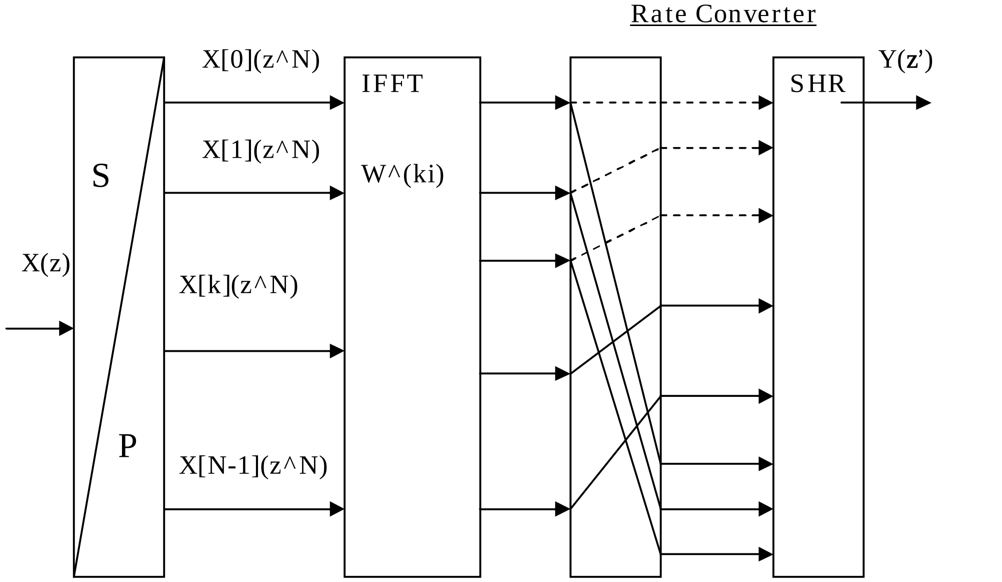
Fig. 4-2 Structure of conventional TMUX de-multiplexer

The great feature of the conventional TMUX was the reduction of processing loads.
Without TMUX each channel needs N.L times multiplication for the filtering, N times for the frequency conversion totaling in N.(L+1). With N channels the total number of complex multiplication is N^2(L+1) for each NT period.
With TMUX the required number of complex multiplication is N.log(N) for FFT and L times for the digital filtering.
Thus 1/(NL) times reduction of signal processing was achieved. Another feature is an exact frequency characteristics design by the digital filter and the identical characteristics for all the channels because the same filter is shared by all the channels.

5. OFDM

OFDM(Orthogonal Frequency Division Multiplex) is widely used for digital broadcasting, mobile communication, wireless LAN and ADSL systems. The signal is S/P (serial to parallel) converted into multiple OFDM channels. The adopted digital filter is no filtering, or sample holding (of 0-th order) for just one sample period.

5.1 Structure of OFDM network



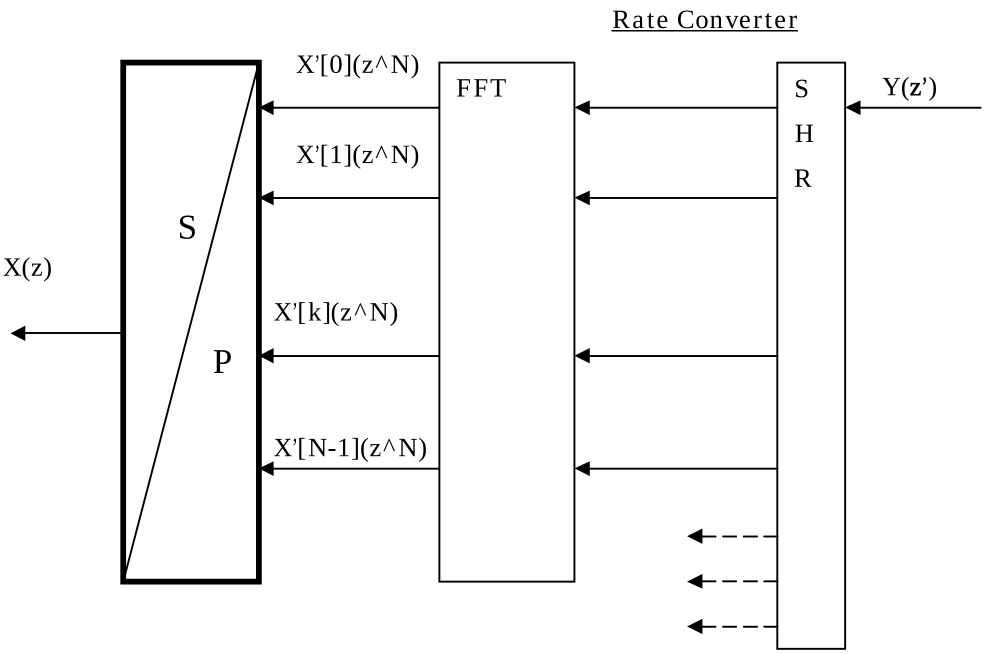


Fig.5-1 Structure of OFGM transmitter and receiver

Rate conversion

Another feature is the rate conversion at the output of the transmitter and the input of the receiver. The rate is expanded at the transmitter and reduced at the receiver. The output Y(z') has the sampling rate T' which is related with T by ; N.T = (N+G).T'. The sampling frequency, which is the multiplexed signal bandwidth is 1/T' = (1 + G/N) /T, Where G is the number of guard samples.

At the receiver the guard samples are discarded and N samples are fed to the FFT circuit. Thus the function of the OFDM is the same as the conventional TMUX.

5.2 Frequency spectrum of OFDM signal

The channel filter for OFDM is

$$G(z) = 1 + z^{-1} + \dots + z^{-(N-1)} = \{1 - z^{-N}\} / \{1-z^{-1}\}$$

In frequency

$$G(j\omega) = e^{-j\omega T(N-1)/2} \cdot \sin(N\omega T/2) / \sin(\omega T/2)$$

The channel frequency characteristics extend beyond the channel bandwidth 1/NT (Hz). In fact all channel overlap in the frequency domain.

Thus the channels are not orthogonally multiplexed in the sense of the conventional TMUX.

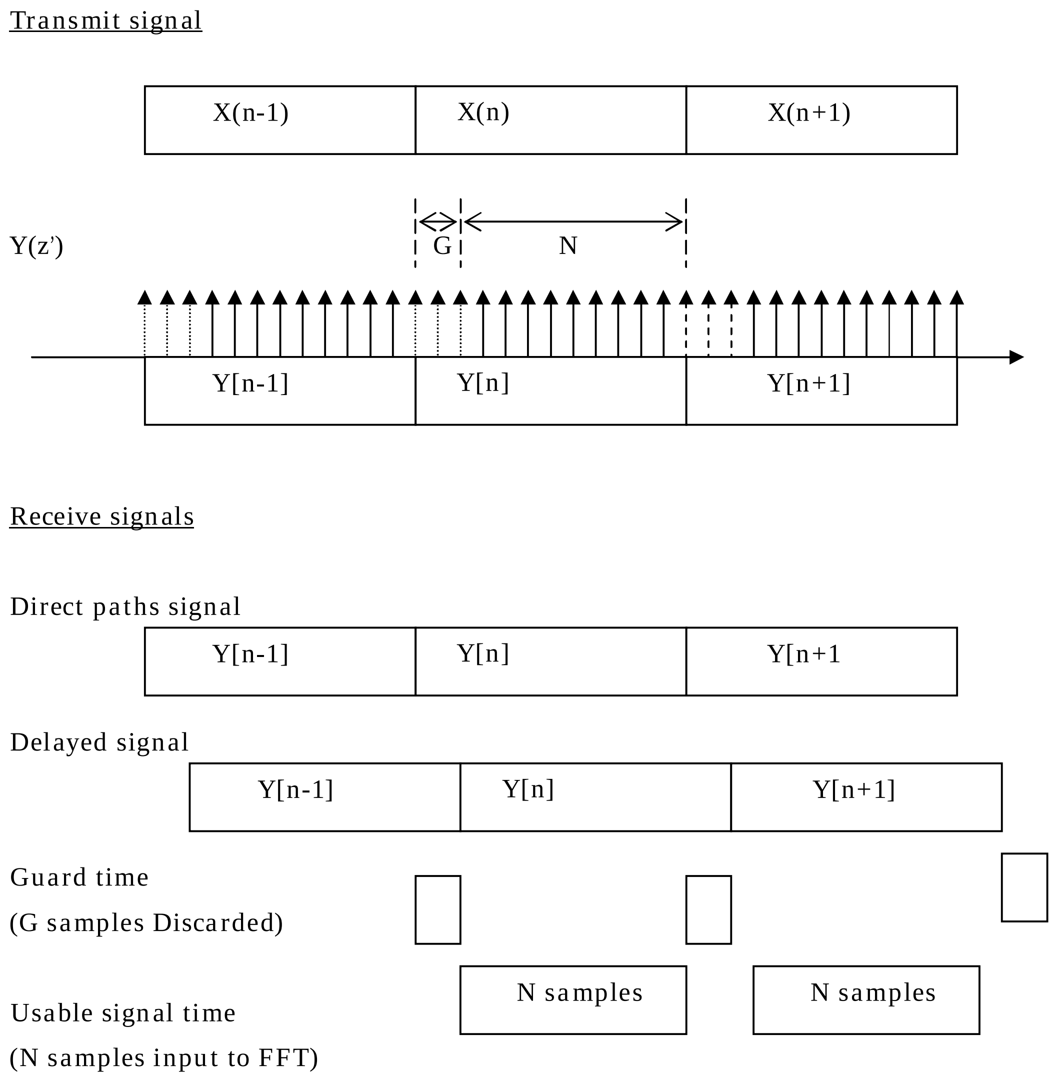
However the total system with the transmitter and receiver combined gives orthogonal channels multiplexing. This is apparent from the IFFT and FFT operations at the TX and RX systems.

5.3 Resilience against multi-path fading

The OFDM is resilient against frequency selective fading since each channel is of very narrow bandwidth hence the fading can be treated as simple fluctuation of the signal amplitudes. The multi-path interferences occur in such a way that when certain channels are cancelled certain other channels are strengthened by phase summation of the direct path and multi-paths signals. Application of error correction coding together with interleaving can realize a reliable communication through such propagation paths.

Another feature is the simple elimination of inter-symbol interferences caused by delay spreads through processing of the guard time. The duration of each channel data is NT. The earlier part of a receive signal can be the sum of the current data and the delayed tails of the previous data. This inter-symbol interference from earlier symbols can be eliminated by simply discarding the G samples appended in the rate conversion at the OFDM transmitter. If the guard time is greater than the delay spreads of the

inter-symbols interferences, then the interferences can be effectively eliminated.
.The processing is depicted in the following figure.



The guard time samples are discarded at the receiver to eliminate the inter-symbol interferences effectively. This is achieved at the cost of expanded RF bandwidth by the ratio; 1+ G / N. The guard time length is G.T' = G.T/(1+G/N) = (N.T).G/(N+G) which must be designed to be greater than the maximum multi-path delays in the system.

6. New applications of TMUX

The OFDM is widely applied to digital broadcasting, mobile communication, wireless LAN and ADSL networks. The narrowband feature of OFDM can be effectively used to suppress the effects of signal reflections and multi-path fading.

The generalized TMUX can provide flexible FDM-TDM converters for communication satellites, base stations of mobile communication networks and other hub stations where a large number of signals concentrate for processing.

The flexibility of channel bandwidth design by the generalized TMUX will find applications beyond communication including digital implementation of chirp filters widely used in Radar systems.

References

[1] M.Bellanger and J.L.Daguet, " TDM-FDM trans multiplexer; Digital polyphase and FFT", IEEE trans. Vol. Com-22, pp.1199-1204, Sept. 1974

[2] R.Maruta and A.Tomozawa, "An improved method for digital SSB-FDM modulation and demodulation ", IEEE. Trans. Vol. Com-26, pp720-725, May, 1978

[3] F.Takahata, Y.Hirata, A.Ogawa and K.Inagaki, "Development of a TDM/FDM Trans Multiplexer", IEEE trans. Vol. Com-26, pp726-733, May,1978

[4] W.H.HYM, C.C.D. Kwan, F.P.Coakley and B.G.Evans, " Multi-carrier demodulators for on-board processing satellites", International Journal of Satellite Communications, Vol.6, pp.243-251, 1988

[5] H.Scheuermann and H.Goekler, " A comprehensive Survey of Digital transmultiplexing Method", Proceedings of IEEE, vol.69,No.11,November,1981

[6] M.Alard and R.Lassalle, " principles of modulation and channel coding for digital broadcasting for mobile receivers", EBU Review technical, No.224, pp168-190 August, 1987

[7] K.Sistanizadeh, et al, "Multi-tone transmission for Asymmetric digital subscriber lines (ADSL)", ICC'93, May 23-26, 1993, Geneva

[8] Osamu Ichiyoshi, "Generalized Transmultiplexer and its application to Mobile Satellite Communications" IMSC90, Ottawa ,IMSC; International Mobile Satellite Conference

[9] O. Ichiyoshi, "Chirp filter" , Japanese Patent Application H1-311715, 1989 (In Japanese)