On Effect of Timing Errors in OFDM/OFDMA

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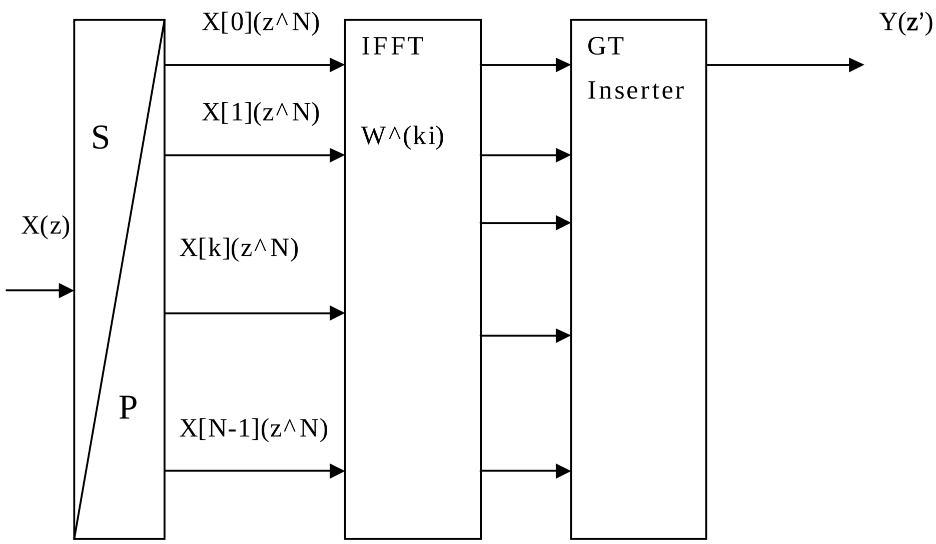
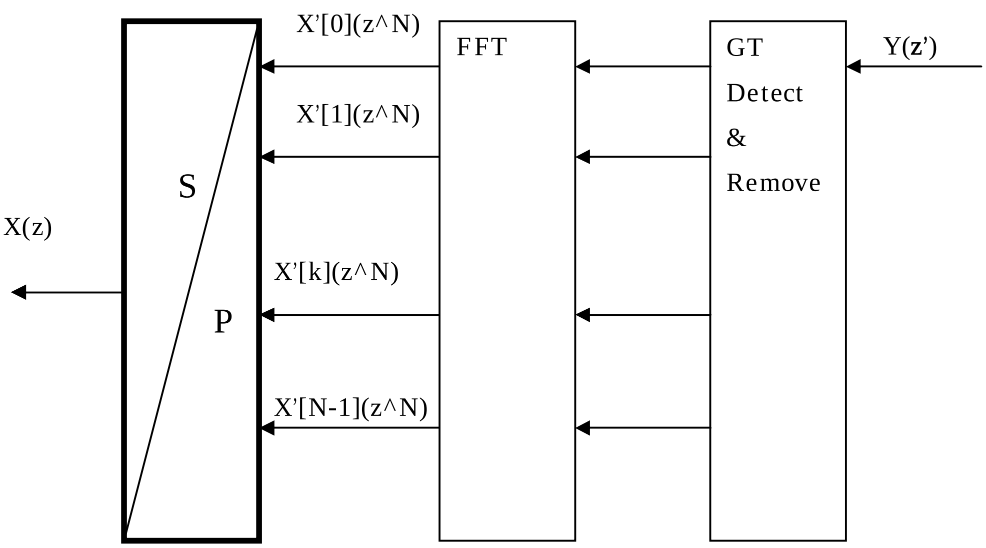
1. FFT implementation of OFDM signals

The most commonly used implementation is based on FFT as in the following figures;

OFDM Transmitter



OFDM Receiver



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2. Receiver of a single channel of OFDM signals

The receive OFDM signals after Guard Time removal is;

$$r(t) = [k = 0, N-1] \sum [n] \sum a[k](t - n.T + \varepsilon [k]) \,.\, e\hat{\ } j\{k. \Delta \omega.(t - n.T + \varepsilon [k])\}$$

where $\Delta \omega = 2\pi/T$

$\varepsilon [k]$; Timing error of the k-th channel

Let $t\text{-}n.T \rightarrow t$

and treat the signal in the n-th time frame;.

$$r(t) = [k = 0, N-1] \sum a[k](t + \varepsilon [k]) \,.\, e\hat{\ } j\{k. \Delta \omega.(t + \varepsilon [k])\}$$

Receive signal sampling

Receive signal sampled at the rate N/T (Hz);

$$r[m] = [k = 0, N-1] \sum a[k](m.T/N + \varepsilon [k]) \,.\, e\hat{\ } j\{k. \Delta \omega.(.m.T/N + \varepsilon [k])\}$$

$$=[k = 0, N-1] \sum \sum a[k](m.T/N + \varepsilon [k]).e\hat{\ }(j.k. \Delta \omega. \varepsilon [k]).W\hat{\ }(-k.m) \qquad (m = 0,1,2,,,N-1)$$

where $W = e\hat{\ }(- j.2\pi/N)$

FFT output;

$$y[k] = [\ m = 0, N-1]\ W\hat{\ }(k.m)\ .r(m)$$

$$= [m, k' = 0, N-1] \sum \sum a[k'](m.T/N + \varepsilon [k'])\ .e\hat{\ }(j.k'. \Delta \omega. \varepsilon [k]).W\hat{\ }((k-k').m)$$

$$= [m, k' = 0, N-1] \sum \sum a[k'](m+x[k']))\ .W\hat{\ }(-k'.x[k']).W\hat{\ }((k-k').m)$$

where $x[k'] = [N. \varepsilon [k'] / T]$ (integer by Gauss symbol)

The timing error causes detection errors when the modulation data changes between the consecutive frames;

$$a[k'](m + x') = a[k'] \qquad (m=0,1,2,,,,N- 1 - x') \qquad (x' = x[k'] \text{ for simplicity})$$

$$= -a[k'] \qquad (m= N-x, N-x+1,,,,,N-1)$$

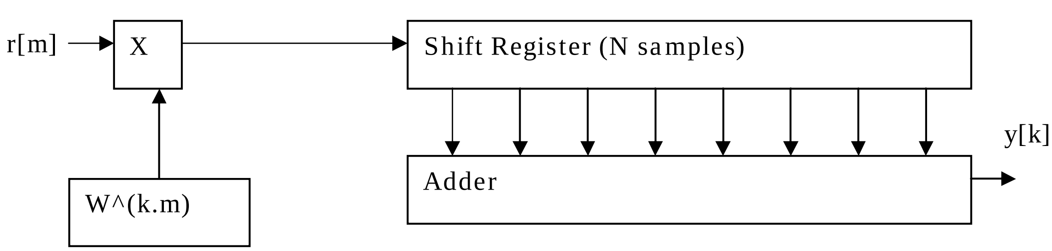
Then the FFT output is;

$$y[k] = a[k].W\hat{\ }(-k.x)\ .N.\ (1- 2x/N)$$

$$+ [k'=/= k] \sum a[k'].W\hat{\ }(-k'.x').W\hat{\ }((k-k')/2).\ \sin\{2\pi/N.(k-k').x'\} / \sin\{\pi/N.(k-k')\}$$

The second terms are the interferences from channels k' to k. Note for $x' = x[k'] = 0$, the interferences from channel k' is zero. Also note for $|(k-k').x'/N| \ll 1$, the magnitude of the interferences are 2x'.

The above processing is equivalent to the following receiver structure;



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3. Interferences coefficient

$$Y[k] = a[k] + [k' =/= k] \sum K(k,k').\, a[k']$$

Where the interferences coefficients are

$$K(k,k') = 1/(N-2x).W\text{^}(k.x - k'.x').W\text{^}((k-k')/2).\ \sin\{2\pi/N.(k-k').x'\} / \sin\{\pi/N.(k-k')\}$$

The coefficients are approximated;

$$|K(k,k')|\ (=)\ 2|x'|/N \qquad (|2\pi/N.(k-k').x'| << 1)$$

$$=< 1/\pi/|k-k'| \qquad (|k-k'|/N << 1)$$

$$=< 1/N \qquad (|k-k'|/N\ (=)\ 1/2)$$

4. Timing Error Correction

Send the same signal a[k] to channels k and k' for frequency diversity.

$$y[k] = a[k].W\text{^}(-k.x)\ .N.\ (1- 2x/N)$$

$$y[k'] = a[k].W\text{^}(-k'.x)\ .N.\ (1- 2x/N)$$

Then conduct

$$Y[k].y[k']^*\ (=)\ |N.a[k]|\text{^}2\ .W\text{^}((k'-k).x) \qquad (|x|/N << 1)$$

Which gives detection of the timing error x.

The detected error can be fed back to the timing generator for timing synchronization; for receive synchronization for the receiver or for transmit synchronization for OFDMA.

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