Adaptive Interference Cancellation; Analysis by Signal Space Theory

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Foreword

Interferences problems get more serious with growth of communication networks. As available frequency and other channel resources (i.e. space) are limited, more channels naturally increase mutual interferences. With best preventive measures, serious interference problems can occur in an unpredictable manner. Therefore the adaptive interferences cancellation technology is essential for the growth and quality of the communication networks.

In this memorandum a unified approach to the adaptive interferences cancellation technology is attempted for deeper understanding and more effective design of the systems.

Contents

1. Signal Space Theory

Suppose a desired signal Sd and a number n of interfering signals; Si (i= 1,2,3,,,n) exist in the system. Those signals are of independent sources, hence mutually uncorrelated. Through the transmission media they are intermingled thus interferences problem occur in many communication networks.

We define some concepts in order to measure "similarities " among different signals.

Correlation;

The correlation between two signals X and Y is defined by the following inner products;

$$(X,Y) = 1/2\pi \int X(\omega).Y^*(\omega)d\omega$$

$$(x,y) = \int x(t) .y^*(t) dt$$

where $X(\omega)$ is the Fourier transform of x(t), i.e. $X(\omega) = \int x(t) .e\text{^}(-j\omega t).dt$

It can be shown that the above two formulae give the same value which is defined as the correlation between signals X and Y.

Orthogonality

The signals Sd, Si (i=1,2,3,,,,n) are from independent sources hence uncorrelated;

$$(Si,Sj) = 0 \qquad (i=/=j)$$

Norm

For the same signals the correlation is the total energy of the signal;

$$(Si,Si) = //Si//\text{^}2 =1/2\pi \int /X(\omega)/\text{^}2d\omega \qquad = \int /x(t)/\text{^}2 dt$$

//Si// is called the norm of signal Si.

The definition can be generalized so that the square of the norm //Si//^2 stands for the power (joules /second) of the signal.

Likelihood between two signals

The correlation between signals X and Y is decomposed into more details;

$$(X,Y) = //X//.//Y//.\cos(\theta[X,Y]).e\text{^}(j\varphi[X,Y])$$

or

$$\cos(\theta[X,Y]).e\text{^}(j\varphi) = (X,Y) / (//X//.//Y//)$$

The amplitude of the above quantity is called likelihood between signals X and Y.

If X and Y are identical, then $\cos(\theta[X,X]) = 1$, or $\theta[X,X]) = 0$. If there is no correlation between X and Y, then $\cos(\theta[X,X]) = 0$, or $\theta[X,X]) = \pi/2$.

$\varphi[X,Y]$ is the phase between X and Y. It is apparent from the definition that

$$\theta[X,Y]) = \theta[Y,X]) .$$

$$\varphi[X,Y] = - \varphi[Y,X]$$

Signal space

Suppose a communication network where exist signals Sd, S1,S2,,,,Sn from original sources. A receiver is to receive the signal Sd but also receives the other interfering signals. The output X of the receiver will be ;

$$X = L_{dd}.S_d + \sum_{i=1}^{n} L_{di}.S_i$$

Without loss of generality we normalize the norm of the original signals;

$$\|S_d\| = \|S_i\| = 1 \quad (i=1,2,3,,,,,n)$$

Then

$$X = (X,S_d).S_d + \sum_{i=1}^{n} (X,S_i).S_i$$

$$= \|X\|.\cos(\theta[X,S_d]).e^{(j\varphi[X,S_d])}.S_d + \|X\|.\sum_{i=1}^{n} \cos(\theta[X,S_i]).e^{(j\varphi[X,S_i])}.S_i$$

or

$$X / \|X\| = \cos(\theta[X,S_d]).e^{(j\varphi[X,S_d])}.S_d + \sum_{i=1}^{n} \cos(\theta[X,S_i]).e^{(j\varphi[X,S_i])}.S_i$$

The norm of the above signal equals 1.

Hence,

$$\cos(\theta[X,S_d])^2 + \sum_{i=1}^{n} \cos(\theta[X,S_i])^2 = 1$$

Namely the square sum of the likelihoods equals 1. This fact suggests that the likelihood of the signals corresponds to the directional cosines in n+1 dimensional Eucledian space.

Exercise

Prove the following formulae;

$$\|X+(-)Y\|^2 = \|X\|^2 + \|Y\|^2 +(-)\, 2\|X\|.\|Y\| \cos(\theta[X,Y]).\cos(\varphi[X,Y])$$

$$\|X+Y\|^2 + \|X-Y\|^2 = 2\,(\|X\|^2 + \|Y\|^2)$$

2. Adaptive Interference Cancellation Systems

Main path and auxiliary paths

We have a main path receiver to receive the desired signal Sd. In reality interference signals Si (i=1,2,,,n) will be also received by the receiver. The output signal X of the main path receiver can then be depicted as

$$X = Sd + [i=1,n]\sum \ Ldi.Si + Nd \qquad (2\text{-}1)$$

Where Ldi is the interference coefficient from Si to X. Nd is thermal and other external noise.

In order to reduce the interferences in X, auxiliary paths Yi (i=1,2,,,m) are set to receive the interference signal Si. The signal Yi is expressed as follows;

$$Yi = Si + Di.Sd + [j=/=i]\sum \ Lij.Sj \ + Ni \qquad (2\text{-}2)$$

If we define Lii=1; then

$$Yi = Di.Sd + [j=1,n]\sum \ Lij.Sj \ + Ni \qquad (2\text{-}3)$$

Where Di denote leakage of signal Sd and Lij that of Sj into Yi.

Ni is the thermal and other external noises.

Interferences cancellation circuit

The auxiliary signals Yi are subtracted from X through weighting factors Wi to give the output Z;

$$Z = X - [i=1,m]\sum \ Wi.Yi$$

$$= \alpha.Sd + Sz + Nz \qquad (2\text{-}4)$$

Where

$$\alpha = 1- \ [i=1,m]\sum \ Wi.Di \qquad (2\text{-}5)$$

$$Sz = [j=1,n]\ [i=1,m]\sum\sum \ (Ldj - Wi.Lij).Sj \qquad (2\text{-}6)$$

$$Nz = Nd - [i=1,m] \ \ Wi.Ni \qquad (2\text{-}7)$$

Sz and Nz are respectively the residual interferences and thermal noise in the cancellation circuit output Z.

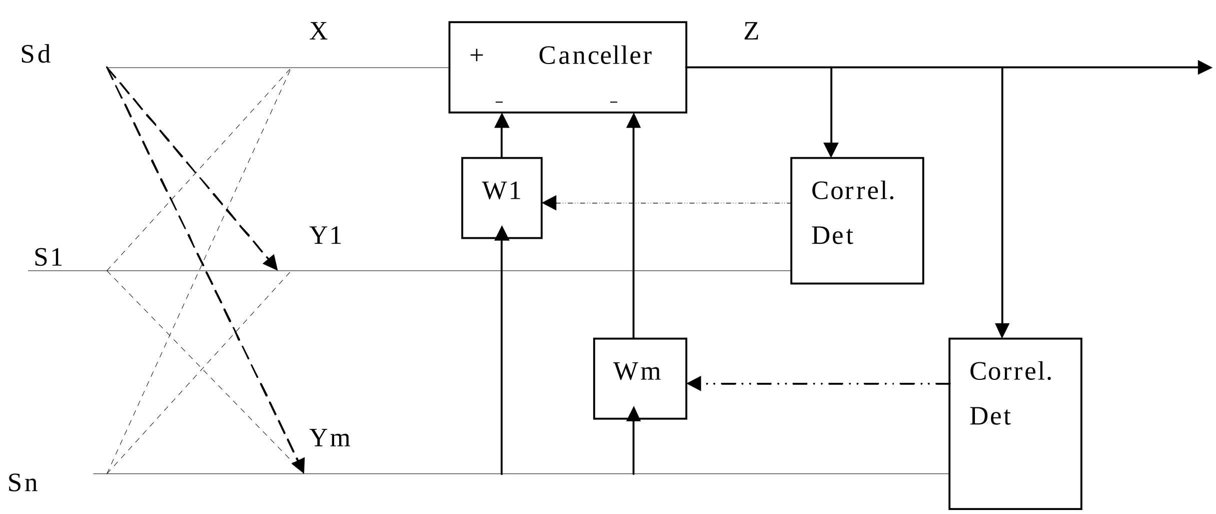


Fig.2-1 Adaptive Interference Cancellation Circuit

3. Least Mean Square Error (LMSE) methods

The task now is to establish how to control the weighting factors Wi (i= 1,2,,,m). An intuitive method is to minimize the power of the cancellation output Z.
We minimize //Z//^2 = (Z, Z) by control of Wi;

Stationary state

A necessary condition for the minimization is that //Z//^2 be a stationary state, i.e.

$$[\partial / \partial Wi]//Z//^2 = 0 \ (i=1,2,,,m) \quad (3\text{-}1)$$

The direct result is;

$$(Z, Yi) = 0 \quad (3\text{-}2)$$

That is, the output of the cancellation circuit is minimized when Z gets orthogonal, or uncorrelated to every auxiliary path signal Yi.

How to control Wi

The correlation (Z,Yi) needs measurement time according to the bandwidth of the observed signals Sd. Hence a natural method is a sample and hold control. The weight Wi[k] is held constant for control period k while the correlation is measured. The weight is then renewed for the next control period according to the following formula;

$$Wi[k+1] = Wi[k] - \gamma .(Z, Yi) \quad (3\text{-}3)$$

Where γ is the loop gain of the control circuit.
The convergence into the correct state is not always guaranteed but if the stationary state is achieved, i.e. Wi[k] = Wi[k+1], then (Z, Yi) = 0.

Performance of LMSE method

In the stationary equilibrium state

$$0 = (Z, Yi)$$
$$= (\alpha .Sd + Sz + Nz , \ \ Di.Sd + [j=1,n]\sum \ Lij.Sj \ \ + Ni)$$
$$= \ \alpha .Di + (Sz, [j=1,n]\sum \ Lij.Sj) - Wi..//Ni//^2 \quad (3\text{-}4)$$

Or

$$(Sz, [j=1,n]\sum \ Lij.Sj) = -\alpha .Di + Wi..//Ni//^2 \quad (3\text{-}5)$$
$$(Sz, [j=1,n]\sum \ Lij.Sj) = \ [j=1,n]\sum \ Lij^*.(Sz,Sj)$$
$$= //Sz//. [j=1,n]\sum \ Lij^*.\cos(\theta[Sz,Si]).e\hat{}(j\phi[Sz,Si])$$
$$= //Sz//.Lzi^* \quad (3\text{-}6)$$

Where

$$Lzi = [j=1,n]\sum \ Lij^*.\cos(\theta[Sz,Si]).e\hat{}(j\phi[Sz,Si]) \quad (3\text{-}7)$$

Lzi is interpreted as redistribution of the interferences signals in Yi to the residual interference components in the output Z.

Here we assume the system is interference dominated or the thermal noise Ni is negligible against the interference signal Sz.

Them taking the absolute square of both sides and summing over i= 1,2,,,,m

$$//Sz//^{2}../Lz/^{2} =/\alpha/^{2}. [i=1,m]\sum /Di/^{2} \qquad (3\text{-}8)$$

Where

$$/Lz/^{2} =[i=1,n]\sum /Lzi/^{2} \qquad (3\text{-}9)$$

We set

$$/Di/^{2} = //DiSd//^{2} \;/\; //Si//^{2} = 1/SIYi \qquad (3\text{-}10)$$

Where SIYi is interpreted as the Signal / Interference power ratio of the auxiliary path Yi where the interference signal Si is desired and the leakage of the desired signal Sd is undesired.

Then the output Signal over Interference Ratio of signal Z denoted as SIZ is;

$$SIZ = //\alpha.Sd//^{2} / //Sz//^{2} = /\alpha/^{2} \;/\; //Sz//^{2} \qquad (3\text{-}11)$$

The inverse relations are simpler;

$$1/SIZ = /Lz/^{2} . [i=1,m]\sum 1/SIYi \qquad (3\text{-}12)$$

If we denote

$$ASIY = 1/ \{ [i=1,m]\sum 1/SIYi \} \qquad (3\text{-}13)$$

Then

$$SIZ = Lz^{(-2)} . ASIY \qquad (3\text{-}14)$$

ASIY is interpreted as the aggregate signal-to-interference power ratio of all the auxiliary paths Yi, i=1,2,,,m.

In the case that there is a single interference S1, then Lz =1, hence

$$SIZ = SIY \qquad (3\text{-}15)$$

Which is apparent from the following signal space diagram.

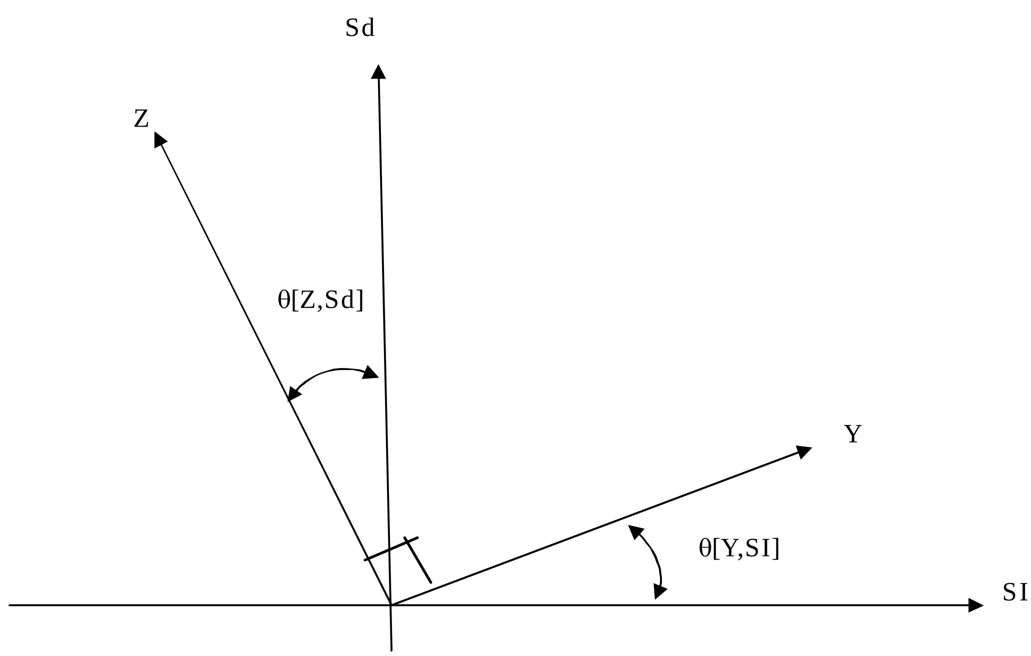


Fig 3-1 Signal space representation of LMSE interferences cancellation performances

4. Decision Feedback Least Mean Square Methods

The above method has serious problems;

(1) The S/I ratio of the cancellation circuit output is at most equal to the S/I of the auxiliary path signals where the wanted signals are the interference signals and the leakage of the desired signal is unwanted.

(2) The leakage of the desired signal Di.Sd into Yi causes the Yi plane to deviate from the interference signal space SI by angle θ[Y,SI] which causes the output signal Z to deviate from the desired signal Sd by the angle θ[Z,Sd]. Since θ[Z,Y] = π/2, naturally . θ[Z,Sd] = θ[Y,SI], or SIZ = SIY as depicted in Fig.3-1.

(3) As the auxiliary paths are usually of inferior selectivity (for example a smaller antenna) than the main path, SIY< SIX, or SIZ< SIX. Thus the above processing will mostly degrade rather than improve the signal to interference powers ratio.

Decision feedback method

The weighting factors Wi are controlled by the correlation detection (Z,Yi) as in eq.(3-3). Since the reduction of the leakage Di of the desired signal Sd into Yi is practically impossible, what if the Sd component in Z is reduced? Then the effect of the leakage Di in the correlation detection can be effectively reduced.

Suppose a replica Sd' of the desired signal Sd is obtained from Z and then the Sd' component is eliminated from Z, then the resultant signal Z' will be; from eq (2-4),

Z' = α'.Sd + Sz + Nz (4-1)

Then the LMSE operation will achieve the following stationary state;

0 = (Z',Yi)

= (α'.Sd + Sz' + Nz' , Di.Sd + [j=1,n]∑ Lij.Sj + Ni)

= α'.Di* + (Sz', [j=1,n]∑ Lij.Sj) – Wi..//Ni//^2 (4-2)

By similar calculation in chapter 3, the resultant SIR ratio will be

SIZ' = { /α./ / /α'./}^2.SIZ = { /α./ / /α'./}^2. Lz^(-2) . ASIY (4-3)

= { /α./ / /α'./}^2. SIY (in the case of a single interference) (4-4)

It is shown that the resultant S/I performance is improved by factor { /α./ / /α'./}^2..

Improvement factor by decision feedback

The process to eliminate Sd component from Z is the same process as the interference cancellation. Suppose Sd' is the replica of Sd obtained from Z.

Sd' = γ.Sd + Sd" (4-5)

Sd" is a signal regeneration noise which is orthogonal to any other signal components.

We will normalize

1 = //Sd'//^2 = γ^2 + δ^2 (4-6)

where

$$\delta\text{^}2 = //Sd''//\text{^}2 \quad (4\text{-}7)$$

Now

$$Z' = Z - V.\ Sd' \quad (4\text{-}8)$$

Is orthogonalized to Sd' by LMSE method.

$$(Z - V.Sd',\ Sd') = 0 \quad (4\text{-}9)$$

which gives,

$$V = \alpha\,.\,\gamma \quad (4\text{-}10)$$

And

$$\alpha' = \alpha\,.(1-\gamma) \quad (4\text{-}11)$$

Thus the improvement factor is

$$\{\ /\alpha\,.///\alpha'./\}\text{^}2. = 1/\ (1-\gamma)\text{^}2 \quad (4\text{-}12)$$

Note the improvement factor can get infinity, or a perfect interferences cancellation can be achieved if $\gamma = 1$, or the desired signal is perfectly regenerated from Z.

<u>How to regenerate the desired signal</u>

(1) Demodulation of digital signal

Demodulated digital signal sequences give a very exact replica of the desired signal when the bit error rates are low. Approximately the improvement ratio will be [Bit Error Rate of DEM]^(-2).

(2) Non-existence of signal

When the signal is of an intermittent nature and the timing of the signal presence is known then the non-existence periods of the signal gives a good time window to perform the correlation detection.

(3) Hard limiting

When the signal amplitude is constant then a simple hard limiter can regenerate the desired signal with improved S/I power ratio up to 6 dB.

Let us assume that a sum of desired signal A.e^(jωc.t) and an interference signal a.e^(jω1.t) is put into a hard limiter. The combined input signal is;

$$A.e\text{^}(j\omega c.t) + a.e\text{^}(j\omega 1.t) = A.\ e\text{^}(j\omega c.t)$$
$$.\sqrt{\{1+2(a/A).\cos(\Delta\omega.t) + (a/A)\text{^}2\}}$$
$$.e\text{^}[j.\arctan\{(a/A).\sin(\Delta\omega.t)/\ (1+2(a/A).\cos(\Delta\omega.t))\}]$$

where $\Delta\omega = \omega 1 - \omega c$

The hard limiter sets the amplitude of the output to constant at any instant. That is, it is equivalent to an adaptive amplifier with amplitude gain;

$$1/\ \sqrt{\{1+2(a/A).\cos(\Delta\omega.t) + (a/A)\text{^}2\}}\ (=)\ 1 - (a/A).\cos(\Delta\omega.t) = 1 - 1/2.(a/A)\{.e\text{^}(j.\Delta\omega.t) - .e\text{^}(-j.\Delta\omega.t)\}$$

When a/A is small, the output of the hard limiter will be

$$Z'\ (=)\ A.e\text{^}(j\omega c.t) + 1/2.a.e\text{^}(j\omega 1.t) - 1/2.a.e\text{^}\{j.(\omega c-\Delta\omega).t\}$$

The first term is the desired signal. The second term is the interference signal and the third the newly generated noise (Sd"). Note the amplitude of the interference is halved. Thus about 6dB improvement can be achieved for the correlation detection performances.

Fig 4.1 shows the improved adaptive interference cancellation circuit characterized as the decision feedback LMSE method. It is a modification from Fig.2-1 with the additional desired signal regeneration and cancellation circuit in the main path and appropriate delays in the auxiliary paths in order to compensate the above processing.

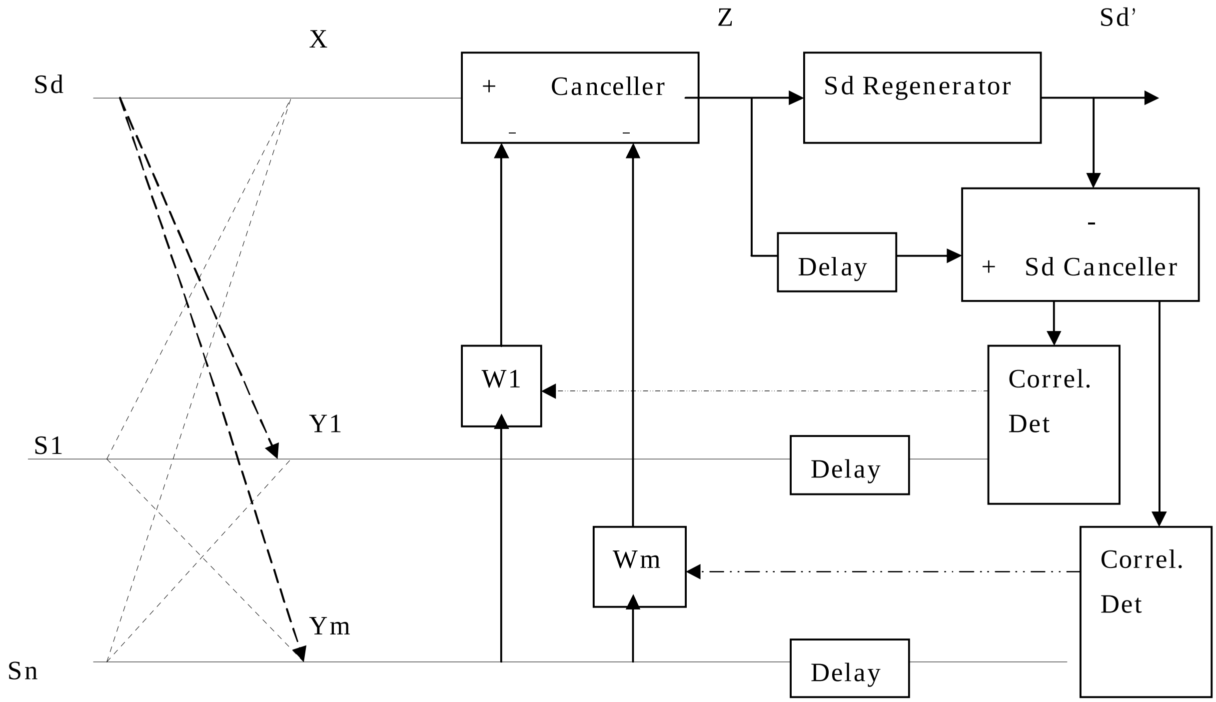


Fig.4-1 General structure of Decision feedback Adaptive Interference Cancellation Circuit

Total DUR

From eq(2-4) to eq(2-7),

Z = α.Sd + Sz + Nz (2-4)

The total power is

//Z //^2 = /α/^2 .//Sd//^2 + //Sz//^2 + // Nz//^2

The output Desired to Undesired power ratio DUZ is

1/DUZ = 1/SIZ + 1/SNZ

where

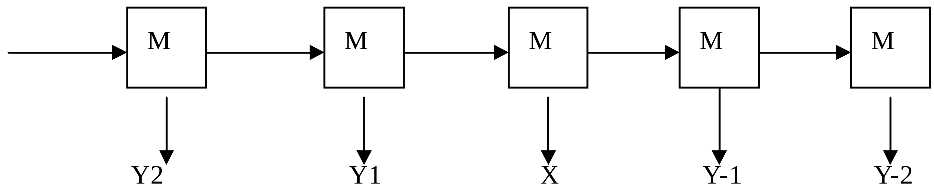
SNZ = (/α/^2) /(// Nz//^2)

The SNR will be degraded by the interference cancellation processing. The thermal noise Ni in Z' and Yi will cause an error in control of the weighting factor Wi. Therefore the interference cancellation is effective only in interference dominated circumstances.

5. Applications of Adaptive Interferences Cancellation Technology

[1] Digital Channel Equalizer

The inter-symbol interferences can be effectively cancelled by the decision feedback equalizer. The receive signal is stored in a shift register. The center tap can give the desired signal path and the forward and backward taps give the auxiliary paths.



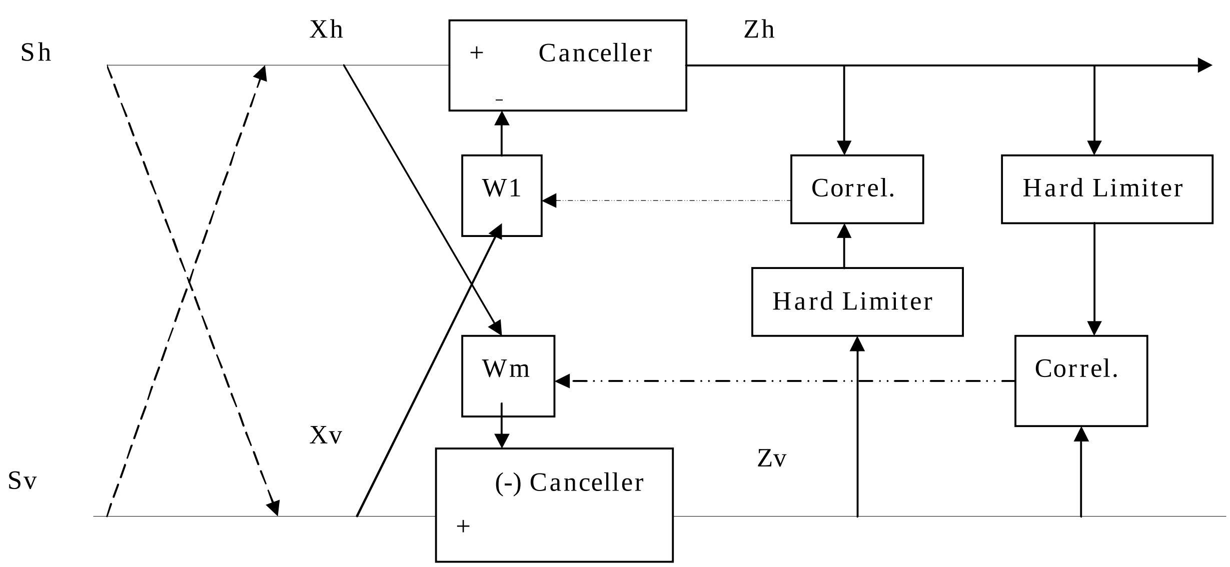
[2] Cross Polarization Interferences Cancellation

The polarization of the radio wave can be effectively utilized to double the frequency bandwidth of the communication channel. It is widely used in microwave and satellite communication networks. For linear polarization Horizontal and Vertical modes are used. For circular polarizations, Right-hand and Left-hand circular modes. At the receiver the Orthogonal Mode transducer (OMT) separates the two polarization signals.

The orthogonality of the polarized signals is degraded in the propagation path due to anisotropic media such as rain drops. The degraded otthogonality (axial ratio) can be effectively recovered by adaptive interference cancellation technology described above in this memoramdum.

The decision feedback LMSE method can be effectively applied.

A unique method is to use simple hard limiters as shown below. The feature is that the SIR improvement is fed-back for further improvement and good recovery of the orthogonality is achieved. A special feature is its generality because no signal regeneration and cancellation is required.



[3] Echo canceller

The echo problem occurs by the signal reflection at the 2W/4W transducer in the distant end. The reflected signal returns to the transmitter and the talker listens one's own voice as echoes from the distant end.

The echo is effectively cancelled by the LMSE method because a faithful replica is available at the transmission point.

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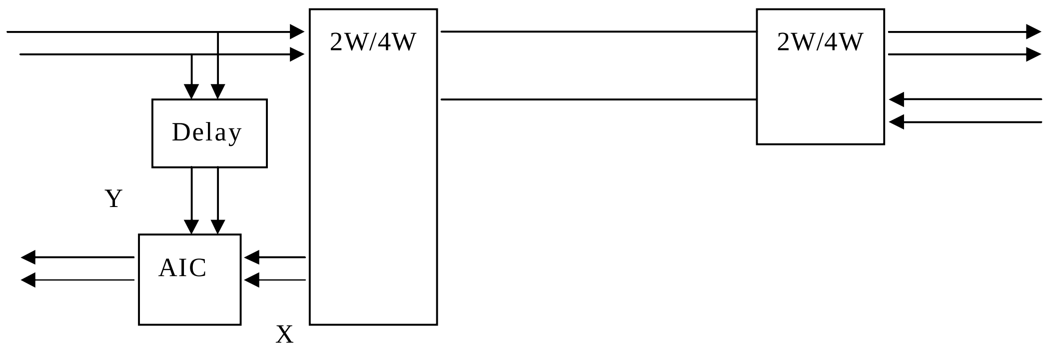

[4] Co-channel Interferences Cancellation

Co-channel interference means interference among signals sharing the same frequency bandwidth. This category includes a large variety of radio communication systems including microwave relay, satellite communication and mobile radio networks. Inter-satellite, inter-bam or inter-sector interferences will be effectively reduced by the Decision Feedback LSME methods.

[5] Environmental interferences cancellation

In airplanes engine noise cancellation is offered to the passengers for better quality of audio channels. For television broadcasting, ghost cancellation is offered to improve the video quality which is degraded by ghosts, or reflection fro the environment.

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