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A Signal Space Theory of Interferences Cancellation Systems

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Human Network for Better 21 Century

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Abstract

Interferences among signals from different sources are universal problems in communication networks. In general the directions, bandwidths, modulation schemes and even numbers of the interferences are unknown at the receiver. The main receiver is set to receive the desired signal but it may also receive an unknown number of interference signals of unknown nature. In order to cancel those interferences we set a number of auxiliary receivers aimed to collect the interference signals. The auxiliary receiver outputs are adaptively weighted in amplitude and phase and are combined with the main receiver output in order to cancel the interference signals therein. The problem is; how can we control the adaptive weights without knowledge about those interference signals? One universal method is Least-Mean-Square-Error (LMSE) method which is based on the belief that the output signal after successful cancellation of the interference signals will give the minimum power output. Although this is quite a reasonable assumption the author showed a fundamental limit exists in the method, which practically deteriorates rather than improve the quality of the output signal. The author also proposed a signal space concept that can graphically show the mechanism of the problem [1].

In this paper the author reports a further study of the signal space theory. A “tangent square summation theorem” gives the basis of the signal space theory. The square tangent of a vector in the signal space corresponds to the inverse of Signal-to-Interference power ratio (SIR), hence the theorem can be restated as “Inverse SIR summation theorem”. The theorem tells the more auxiliary paths signals bring the greater SIR deterioration of the output from the canceller based on LSME algorithm. If the number of the auxiliary paths exceeds the number of the interferences signals, we will have simply a zero output. This “trivial zero output problem” is readily explained by the signal space theory. The basis theorem is applied to generalization of the theory to include the thermal noise additive to each auxiliary path receive signal.

The problem of the LMSE cancellation method can be solved by elimination of the desired signal component from the correlation measurements for control of the adaptive weights. The essence of the method lies in regeneration of the desired signal, which is the very objective of communication. The mechanism of the improvement is clearly depicted by the Signal Space theory. A few examples are given in the paper.

Keywords

Interferences, LMSE, Decision Feedback, Antenna Side-lobe, Correlation, Uncorrelated, Orthogonality, Originality, Signal Space、Hermite matrix, Hard limiter, Likelihood, Demodulators

1. Signals and Signal Space

Inner Product or Correlation;

Suppose we have two signals S1(t) and S2(t). Then we can define the **inner product** of those signals;

$$(S1(t), S2(t)) = [-1/2T,+1/2T] \int S1(t) \cdot S2^*(t)\, dt / T \quad (T \rightarrow \infty)$$

Where S2(t)* means the complex conjugate of S2(t)

The above inner products are also called **correlation** of the signals S1(t) and S2(t).

Power of signals;

The self correlation of a signal S(t) is physically the power of the signal;

$$(S(t), S(t)) = || S ||^2$$

where || S || is called the **norm** of the signal S(t).

Schwarz inequality;

Suppose we have two signals X(t) and Y(t). Then the correlation of X(t), Y(t) meets the Schwarz inequality;

$$| (X, Y)| = \text{or} < ||X|| \cdot ||Y||$$

Angle Between Signals in Signal Space;

The correlation or inner product between two signals X and Y can be expressed as follows;

$$(X, Y) / (||X|| \cdot ||Y||) = \cos(\theta) \cdot e^{(j\phi)}$$

where θ is the **angle** between vectors X and Y in the Signal Space and ϕ is the phase of the complex value (X,Y).

The amplitude of the above formula;

$$\cos(\theta) = | (X, Y) / (||X|| \cdot ||Y||) |$$

is also called the **likelihood** of signals X and Y.

For $\theta = 0$, the signals are identical; X = Y or totally correlated.

For $\theta = \pi/2$, $\cos(\theta) = | (X, Y) / (||X|| \cdot ||Y||) | = 0$, the signals X and Y are totally **uncorrelated** or mutually **orthogonal** in the Signal Space.

Signal Space;

Suppose we have signals Sd, S1, S2, ,,, Sm from different sources. Then they form a signal space with each signal giving the bases of the space. Without loss of generality, we can normalize their amplitude to 1. ||Si ||=1 for all i.

Here we define **originality** and **orthogonally** of the signals. If two signals S1 and S2 are generated from different sources, then they are **original** and mutually **orthogonal.** Originality → Orthogonality.

Note the inverse is not necessarily true.

The above signal space is a vector space spanned by the original signals {Si ; i = 1,2,3,,,,m}.

Any signal in the communication system is a combination of those signals originating from different sources.

Suppose

$$X = x1 \cdot S1 + x2 \cdot S2 + ,,, xm \cdot Sm$$

$$Y = y1 \cdot S1 + y2 \cdot S2 + ,,, ym \cdot Sm$$

Then

$$(X,Y) = x1 \cdot y1^* + x2 \cdot y2^* +,,,,, xm \cdot ym^*$$

Thus the signals X and Y can be expressed as vectors in the signal space;

$$X = (x1, x2, x3, ,,,,, xm)$$

$$Y = (y1, y2, y3 ,,,,, ym)$$

2. Interference cancellation system

Suppose we have a desired signal Sd to receive and regenerate for communication. There are also other signals S1,S2,,,,Sm generated by different sources that leak into the receive circuit. In order to cancel those interferences, we set a number of auxiliary receivers. Let us denote the main receiver by X that is to receive the desired signal Sd, and the auxiliary receivers Y1,Y2,,,Yn to receive the interferences signals.

The main path and auxiliary paths signals are combinations of those signals;

$$X = Sd \quad + \quad I1 \cdot S1 + I2 \cdot S2 \quad +,,,,+ Im \cdot Sm$$

$$Yi = Di \cdot Sd + Li1 \cdot S1 + Li2 \cdot S2 +,,,, + Lim \cdot Sm \quad (i = 1,2,,,,,n)$$

where Sd and {Sj ; j = 1,2,,,m} are original signals. Without loss of generality we assume the norms of original signals are normalized: ||S||=1.

The {Ii, Lij: i=1,2,,,n, j=1,2,,,,m} are transmit coefficients of the communication paths.

Least Mean Square Error Method (LMSE)

In order to cancel the interference signals, we subtract a combination of the auxiliary paths signals with adaptive weights to get the compensated signal Z.

$$Z = X – [i=1,n] \Sigma\, Wi \cdot Yi$$

where {Wi; I = 1,2,,,,n} are the adaptive weights.

We assume the power of signal Z will be minimal when the intended interferences cancellation is achieved. We control the weights Wi (I = 1,2,,,,n) to minimize ||Z||^2.

To do that we set the partial derivatives of ||Z||^2 by Wi*.

$$\partial ||Z||^2 / \partial Wi^* = 0$$

Then we get;

$$(Z, Yi) = 0 \quad (i = 1,2,,,,n)$$

That is, the output signal must be orthogonal to all the auxiliary paths signals.

The weights {Wi} can be derived from the equation.

$$[k=1,n]\, \Sigma\, (Yk, Yi) \cdot Wk = (X, Yi) \quad (i = 1,2,,,,n)$$

The equations can be expressed more simply;

$$[(Yk,Yi)] \cdot [Wk> = [(X,Yi)> \quad (k, i = 1,2,,,, n)$$

where [(Yk,Yi)] is an n x n matrix with (Yk,Yi) as its (k,i) elements and [Wk> a column (vertical) vector with Wk as the k-th element.

Note the [(Yk,Yi)] is an Hermite matrix;

$$[(Yk,Yi)] = [(Yi,Yk)]^*$$

3. Signal Space Analysis of LMSE Operations

We will now deal with the simplest case where we have only a desired signal Sd and an interference signal S1.

$$X = Sd + I \cdot S1$$
$$Y = D \cdot Sd + L \cdot S1$$

Then the canceller output

$$Z = X - W \cdot Y$$
$$= (1- W \cdot D)Sd + (I - W \cdot L) \cdot S1$$

From $(Z, Y) = 0$,

we get

$$W = (D^* + L^* \cdot I) \ / \ (|D|^2 + |L|^2)$$

where we used ||Sd||^2 = ||S1||^2 = 1.

And the output is

$$Z = A \cdot (L^* \cdot Sd - D^* \cdot S1)$$

where $A = (L - D \cdot I) \ / \ (|D|^2 + |L|^2)$

Let us denote the Signal-to-Interference Power Ratios (**SIR**) of the main, auxiliary and the output signals as

$$SIX = ||Sd||^2 \ / \ || I \cdot S1||^2 = 1 \ / \ | I |^2$$
$$SIY = || L \cdot S1||^2 \ / \ || D \cdot Sd||^2 = | L|^2 / | D|^2$$

Note the objective of the auxiliary path is to collect the replica of the interference signal S1, hence it is the desired signal for the path.

Then the SIR of the resultant output Z is

$$SIZ = || L^* \cdot Sd||^2 \ / \ ||D^* \cdot S1||^2 = | L|^2 / | D|^2$$

An interesting fact is

$$SIZ = SIY$$

regardless of the main path signal SIX.

Since the main path antenna is usually larger than that of the auxiliary path with greater directivity, the above fact means the LMSE processing will rather degrade than improve the SIR performances of the system.

The above adverse effect can be clearly understood by the signal space theory as follows.

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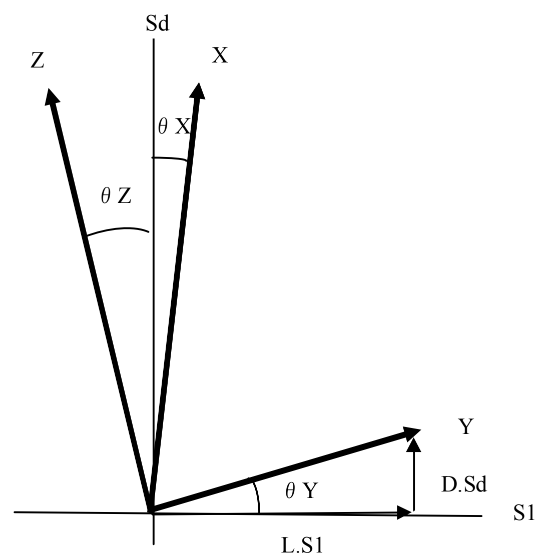


The signal space is defined by unit vectors Sd and S1 which are mutually orthogonal because they come from different origins.

Since Z must be orthogonal to Y, the angles in the above figure are equal; $\theta Y = \theta Z$.

Those angles are related with the SIR of the signals by the formula;

$$\tan^2(\theta Y) = 1 / SIY$$

Since $\theta Y = \theta Z$, hence SIZ = SIY.

4. Tangent Square Summation Theorem

As the output Z must be orthogonal to all auxiliary path signals {Yi} it must be orthogonal to the subspace spanned by those auxiliary paths signals.

$Y_i = D_i \cdot S_d + L_{i1} \cdot S_1 + L_{i2} \cdot S_2 +,,,, + L_{im} \cdot S_m$

$(i = 1,2,,,,,n)$

Case of ideal auxiliary paths receivers;

We first analyze an ideal case that the number of the auxiliary receivers is the same as the number of interferences signals; n = m, and each auxiliary path picks purely the targeted interference signal.

$Y_i = D_i \cdot S_d + S_i$

$(i = 1,2,,,,,m)$

The subspace spanned by the auxiliary path signals is

$Y = [i=1,m] \Sigma\, w_i \cdot Y_i$

$= ([i=1,m] \Sigma\, w_i \cdot D_i).S_d + [i=1,m] \Sigma\, w_i \cdot S_i$

The tangent square of Y is given by

$\tan^2(\theta Y) = | [i] \Sigma\, w_i \cdot D_i |^2 / ([i] \Sigma\, | w_i |^2)$

(To be maximized by wi; i = 1,2,,,,m)

Note $\| S_d \| = \| S_i \| = 1$

By setting

$\partial / \partial w_i^* = 0 \quad (i=1,2,,,,m)$

We get

$D_i^* \cdot ([i] \Sigma\, | w_i |^2) - w_i \cdot ([i] \Sigma\, w_i \cdot D_i) = 0$

Or

$w_i / D_i^* = ([i] \Sigma\, | w_i |^2) / ([i] \Sigma\, w_i \cdot D_i)$

$(i = 1,2,,,,m)$

They must be all equal to a common value, say K

$w_i / D_i^* = K \quad (i=1,2,,,,m)$

Which gives

$\mathbf{\tan^2(\theta Y)} = [i=1,m] \Sigma\, | D_i |^2$

$\mathbf{= [i=1,m] \Sigma\, \tan^2(\theta Y_i)}$

Note

$\tan^2(\theta Y_i) = \| D_i \cdot S_d \|^2 / \| S_i \|^2$

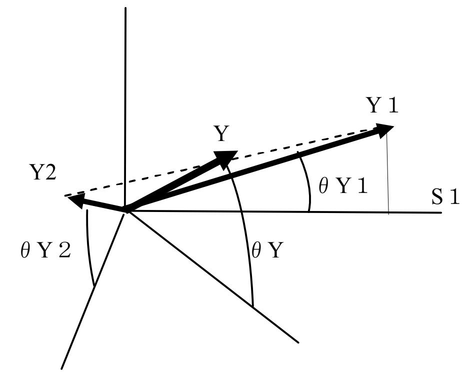
$= | D_i |^2$

The objective of the auxiliary paths receivers is to collect the interferences signals, the leakage of the desired signal component Sd therein is undesired. Hence the signal to interferences power ratio is inverse of the above tangent values. Thus the above tangent square summation theorem can be restated as inverse SIR summation problem.

$\mathbf{1/SIY} = \tan^2(\theta Y) = [i=1,m] \Sigma\, \tan^2(\theta Y_i)$

$\mathbf{= [i=1,m] \Sigma\, 1/SIYi}$

The tangent square summation (TSS) theorem in the special case is depicted in the following figure.



Tangent Square Summation Theorem

Cases in general;

We have the following situation;

$Y_i = D_i \cdot S_d + L_{i1} \cdot S_1 + L_{i2} \cdot S_2 +,,,, + L_{im} \cdot S_m$

$(i = 1,2,,,,,n)$

In vector forms;

[Y> = [D> ・ Sd + [L][S>

Where [Y>, [D>, [S> are column vectors with the i-th elements are respectively Yi, Di, Si. And [L] is the matrix whose (i,m) component is Lim.

If [L] is **regular**, the above equation is applied with the inverse matrix [L^-1],

[Y'> = [L^-1] ・ [Y>

= [D'> ・ Sd + [S>

Where

[L^-1] ・ [D> = [D'>

Or

[L] ・ [D'> = [D>

The above situation is now the same as the special case which tells;

$\mathbf{\tan^2(\theta Y')} = [i=1,m] \Sigma\, | D_i' |^2$

$\mathbf{= [i=1,m] \Sigma\, \tan^2(\theta Y_i')}$

The above operations are linear combinations of the auxiliary paths vectors, which do not alter the structure of the signal subspace {Yi'} = {Yi}, hence

$\mathbf{\tan^2(\theta Y) = \tan^2(\theta Y')}$

5. Representative vector of the auxiliary subspace;

The auxiliary paths signals {Yi; i=1,2,,,,n} form vectors in the signal space {Sd, {Sj}}.
Two vectors Yi and Yj span a plane in the signal space by a linear combination;

$\alpha \cdot Yi + \beta \cdot Yj$

where α, β are arbitrary complex numbers.
The spanned plane is represented by the vector

$Y(i, j) = \alpha(i, j) \cdot Yi + \beta(i, j) \cdot Yj$

which maximizes the power ratio of the Sd component to that of the {Si} components[1].
The above process is repeated to get the vector Y(1,2,,,n) which represents the signal space formed by the auxiliary path signals.
Thus the subspace of the auxiliary paths receivers can be represented by a single vector as if a one-dimensional subspace.

6. Trivial Zero Output Problem

The tangent square summation (TSS) theorem tells

(1) The SIR of the auxiliary subspace monotonically degrades as the number of auxiliary path signals increases.

(2) If the number of the auxiliary paths gets larger than the number of the interferences signals in the system, then the output of the interferences cancellation circuit is a trivial zero.

(3) The mechanism of the problem is evident from the signal space theory. Controlled by LMSE method, the output Z must be orthogonal to all auxiliary path signals Y1.Y2,,,Yn in the signal space which is m-dimensional. If n > m, there can be no non-zero vector Z orthogonal to all auxiliary paths signals; more vectors than the dimension of the signal space.

7. Generalization of the Signal Space

The main path signal X generally contains the desired signal Sd, the interference signals S1,S2,,,,Sm and the thermal noise Nx.
The auxiliary paths signals {Y1,Y2,,,Yn} contain the above signals and additionally the other external signals S(m+1),S(m+2),,, not contained in the main path signal X.
Each auxiliary receive signal Yi also contains thermal noise Ni originated at the receiver antenna and the front head Low Noise Amplifiers (LNA).
The above external signals are uncorrelated with any signals contained in the main path signal X, hence will be included in the thermal noise Ni just for simplicity.
The main path and auxiliary paths signals are now expressed as follows;

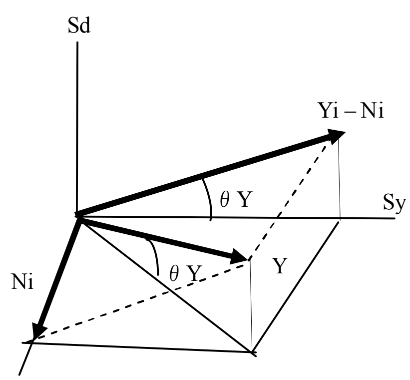
$$X = Sd + I1 \cdot S1 + I2 \cdot S2 +,,,,+ Im \cdot Sm + Nx$$

$$Yi = Di \cdot Sd + Li1 \cdot S1 + Li2 \cdot S2 +,,,, + Lim \cdot Sm + Ni \quad (i = 1,2,,,,,n)$$

How do we incorporate the thermal noise Ni into the signal space?
This problem is readily solved by TSS theorem as follows.
The signal Yi is decomposed into two components;

$$Yi = (Yi - Ni) + Ni$$

The first term contains all the signals Sd, S1, S2,,,,Sm and the second term only the thermal noise. The thermal noise term contains no Sd, hence its tangent angle is zero. Then by TSS theorem the tangent angle of Yi-Ni is equal to that of Yi. In another words the additional thermal and external noises expand the dimensions of the signal space but do not add any tangent square value of the auxiliary paths signal space.
The above description is depicted in the following figure.



Inclusion of thermal and external noises

The external and thermal noises expand the dimension of the signal space without degrading the SIR of the signal space, hence contribute to the stability of the system to avoid the trivial zero output problem. However, they also introduce extra noises to be evaluated by signal-to-noise power ratio; SNR.

8. Improved LMSE

The above analysis tells that the fundamental problem of LMSE method can be solved by elimination of the desired signal component in the correlation measurement to control the adaptive weights {Wi; i=1,2,,,n}.

It is essential to regenerate the replica of the desired signal, which is the very objective of communication.

In digital communications the desired signal can be regenerated at the receiver by demodulation with a good likelihood if the SIR and SNR are sufficiently high. Then the regenerated desired signal replica can be used to remove the desired signal component from the output Z before the correlation measurement. This method is called **decision-feedback**, has been widely used in digital communications.

A simple analysis follows to show the mechanism of the improvement. The symbols <A], [B>, [C] respectively stand for the row vector, column vector and matrix.

$$X = Sd + <I] \cdot [S>$$

$$[Y> = [D> \cdot Sd + [L] \cdot [S>$$

Then the canceller output

$$Z = X - <W] \cdot [Y>$$

$$= (1 - <W \cdot D>) Sd + (<I) - < W \cdot L]) \cdot [S>$$

From Z we regenerate a replica of the desired signal Sd' and subtract it from Z.

Let

$$Sd - Sd' = \varepsilon . Sd \qquad (| \varepsilon | << 1)$$

Then we get

$$Z' = (1 - <W \cdot D>) \varepsilon Sd + (I - <W \cdot L]) \cdot [S>$$

Now the correlation measurement is made between Z' and Y to control the adaptive weights {Wi} to achieve .

$$(Z', Y) = 0$$

Let

$$[Y'> = [D> \cdot \varepsilon Sd + [L] \cdot [S>$$

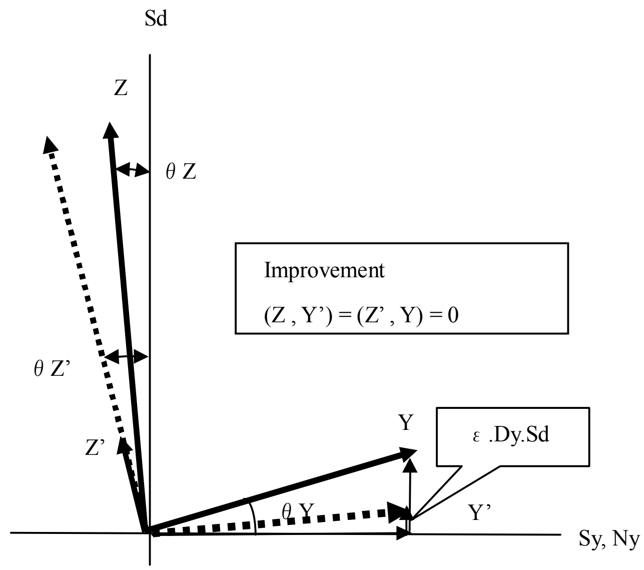
Then, the following equivalence relation holds in the correlation measurement;

$$(Z', Y) = (Z, Y')$$

Thus SIR of output Z shall be improved by $1/\varepsilon$^2 times.

$$SIZ = SIY / \varepsilon \hat{} 2$$

The mechanism of the improvement is depicted in the following figure.



SIR improvement by Decision Feedback Method

5. Conclusion

The Signal Space theory of signals in interferences cancellation system was established for general dimensions (number of the interferences signals) by the proposed Tangent Square Summation (TSS) theorem. It was shown that the TSS theorem is equivalent to Inverse SIR Summation theorem. According to the theorem the SIR of the output of the system only degrades with increase of the number of the auxiliary paths receivers. If the number of the auxiliary paths exceeds the dimension of the signal space, the system gives a trivial zero output. The TSS theorem is used to expand the Signal Space to include the thermal noise added at the receivers. It was shown that the additive noise mitigate the trivial zero output problem at the cost of degraded SNR.

References

[1] Osamu Ichiyoshi " A Signal Space Analysis of Interferences cancellation Systems"
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