

1. Exchange economy

Sale;

The dominant portion of human economy today is based on exchange. The products are sold in the market to get money to continue the cycle of production. The business process of the i -th producer is described by the following equation.

$$s(i) = r(i) + m(i) + w(i) + p(i) + \gamma \cdot (s(i) - r(i))$$

Where $s(i)$, $r(i)$, $m(i)$, $w(i)$ and $p(i)$ are amount of the product sales, the cost for the raw material, the cost for the means of production, the wage for the employed workers and the profit for the employer. Γ is the consumer tax ratio.

As a product sold in a stage is the raw material of the next stage;

$$s(i) = r(i+1)$$

Income;

The incomes of the employers and employees are

$$w(i) + p(i) + t(i) = s(i) - r(i) - m(i)$$

where $t(i) = \gamma \cdot (s(i) - r(i))$ is the consumer tax.

Whole Income and Consumer Tax of the nation;

If we add the above equation over all products exchanged in the market we get the following equation.

$$W + P + T = S - R - M$$

Where S , R and M are the total amounts of the end product sales, the original raw materials and all means of production (machines, factories or any other tools) respectively.

$T = \gamma \cdot (S - R)$ is the consumer tax.

Net income relations;

The R and M are also the products obtained in the market hence parts of S . Let the ratio of those amount $r = R/S$ and $m = M/S$, then

$$W + P + T = (1 - r - m) \cdot S$$

2. Income and expense model

Basic model

Let the income, expense and saving of an economic body i denoted by $y(i)$, $x(i)$ and $z(i)$. Then

$$z(i) = y(i) - x(i)$$

Let us define the gain $g(i)$ of the body by

$$g(i) = y(i) / x(i)$$

then,

$$z(i) = (g(i) - 1) \cdot x(i)$$

Generalization of the model:

Let us now look into the above quantities in the flow of time t . The source of income $y(t)$ is the expense $x(t)$, i.e. investment on the means of production & exchanges, science & technology and the education of the people.

Then the gain is generalized to a function of t ; $g(t)$ which tells the relations between $x(t)$ and $y(t)$ by the formula.

$$y(t) = \int_{-\infty, t} g(\tau) \cdot x(t - \tau) d\tau$$

It tells the income $y(t)$ is the summation of the past expenses $x(t - \tau)$ multiplied by a factor $g(\tau)$ which is called "transfer function".

Let the Laplace transform of $x(t)$, $y(t)$, $g(t)$ and $z(t)$ be $X(s)$, $Y(s)$, $G(s)$ and $Z(s)$;

$$X(s) = \int_{-\infty, t} x(t) \cdot e^{-s \cdot t} dt$$

Then the above relations can be expressed in the following simple form:

$$Y(s) = G(s) \cdot X(s)$$

$$Z(s) = (G(s) - 1) \cdot X(s)$$

Some examples of transfer functions;

Complete squanderer;

This is a case for a person who spends all his income for nothing useful, makes no investment for the future.

$$g(t) = \delta(t) = \begin{cases} \infty & (t=0) \\ 0 & (t > 0) \end{cases}$$

Then,

$$G(s) = 1$$

$$Y(s) = X(s)$$

$$Z(s) = Y(s) - X(s) = 0$$

Constructive economic body

The user spends a portion $a (< 1)$ of his expense on something constructive with the effect lasting in time. Let the measure of time effectiveness by "time constant" tc .

$$g(t) = a \cdot e^{-t / tc}$$

$$G(s) = a / (s + 1/tc)$$

Saving by a constructive economic body;

For simplicity we assume the expense of the body is constant.

$$x(t) = x_0$$

And $X(s) = x_0 / s$

Then the income of the body is given by,

$$Y(s) = G(s) \cdot X(s) = x_0 \cdot a / \{s \cdot (s + 1/tc)\}$$

The inverse Laplace transformation gives

$$\begin{aligned} g(t) = y(t) / x_0 &= a \cdot tc - a \cdot tc \cdot e^{-(t/tc)} \\ &= 0 && (t=0) \\ &= a \cdot tc && (t \gg tc) \end{aligned}$$

The above equations tell the income gain (income) is initially zero because the investment takes time to bear fruits. The transient part exponentially decreases and the income gain reaches the steady state income gain $g_i = a \cdot tc$; investment ratio & time constant product after elapse of time sufficiently greater the time constant.

The saving function is

$$\begin{aligned} z(t) / x_0 = g(t) - 1 &= a \cdot tc - a \cdot tc \cdot e^{-(t/tc)} - 1 = g_i - g_i \cdot e^{-(t/tc)} - 1 \\ &= -1 && (t=0) \\ &= g_i - 1 = a \cdot tc - 1 && (t \gg tc) \end{aligned}$$

The above equation tells;

- (1) Initially the saving factor is -1. With flow of time, the effect of the investment grows and eventually reaches the steady state saving gain $g_s = g_i - 1$ where $g_i = a \cdot tc$.
- (2) If $g_i = a \cdot tc > 1$, then the eventual saving factor is positive and the wealth of the economic body increases with time.
- (3) If $g_i = a \cdot tc < 1$, then eventual saving factor is negative and the wealth of the economic body decreases with time.

3. Growth of the national wealth in transfer function concept

The whole economy of the human society can be directly generalized from the above simple models. A thrifty person saves money by proper management of his/her home economy. The saving is deposited in his/her bank account which the bank uses for investment. Therefore the whole economy can be analyzed by the above transfer function model. The x , y and z can be replaced by X , Y , Z ; the whole expense, income and saving of the society.

The basic equations for the social economy are;

$$\text{Eq-1 ; } \quad W + P + T = S - R - M \quad (\text{Exchange economy model})$$

$$\begin{aligned} \text{Eq-2 ; } \quad Y(t) / X_0 &= a \cdot tc - a \cdot tc \cdot e^{-(t/tc)} && (\text{Transfer function or growth model}) \\ &= 0 && (t=0) \\ &= a \cdot tc && (t \gg tc) \end{aligned}$$

The income ($W+P$) and the consumer tax is spent in the social economy through consumer spending, government spending based on taxation, or business loans through the banks. Therefore,

$$\text{Eq-3 ; } \quad X = W + P + T \quad (\text{Source of expense model})$$

$$\text{Eq-4 ; } \quad S = g \cdot (W+P+T) \quad ; \quad g = a \cdot tc \quad (g: \text{gain of growth})$$

4. Mechanism of GNP growth

The total Income I of the Nation is also the total expense E of the nation.

$$W + P = I ; \text{ for home and industry}$$

$$T \quad ; \text{ Consumer tax for the government}$$

The national expense E is

$$E = I + T = E_c + E_g + E_i + E_s + E_t = E.(e_c + e_g + e_i + e_s + e_t)$$

Where

$$E_c = \text{Expense for consumption}$$

$$E_g = \text{Expense through the government; tax for public functions (administrations, social infrastructure, defense, health care, pensions, etc.)}$$

$$E_i = \text{Investment}$$

$$E_s = \text{Saving}$$

$$E_t = \text{Expense of consumer tax}$$

Economic Growth Model

The above expenses contribute to the growth of GNP; S for the future (in next time unit, a year).

Let $S(n)$ be the GNP in the n -th year. The source of GNP in the next year $S(n+1)$ is the national expense in the previous year $E(n)$. Since $E(n) < S(n)$, the expense must be amplified to bear the GNP for the next year; $S(n+1)$. Let $\langle g \rangle$ be the amplification gain, so that

$$S(n+1) = \langle g \rangle . E(n) = \langle g \rangle . (1 - r - m) . S(n)$$

Or

$$S(n+1) / S(n) = \langle g \rangle . (1 - r - m)$$

For the economy to grow, $S(n+1) / S(n) > 1$, or

$$\langle g \rangle > 1 / (1 - r - m)$$

The aggregate gain $\langle g \rangle$ is given by;

$$\langle g \rangle = e_c + g[g].e_g + g[i].e_i + g[t].e_t$$

Note the coefficient for consumption portion e_c is 1. The coefficients $g[g]$ through the government activities and $g[i]$ through private investment can vary depending on the performances of those activities. $g[t]$ is the gain for the expense of the consumer tax. As the role of the government is to provide basic services for the people and not making profit, $g[g]$ and $g[t]$ are generally smaller than 1. Therefore it should be $g[i]$ the efficiency of development in the private sector that creates the growth of the national economy.

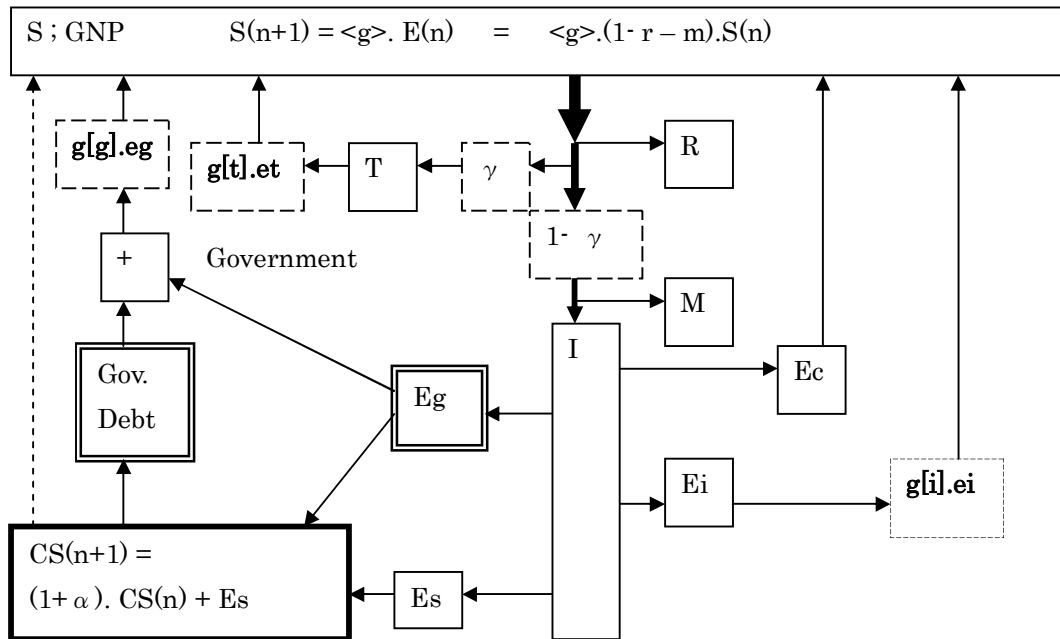
Growth of cash deposits

The remaining portion E_s , the saving part does not contribute to the growth of GNP. But it is added to the cash stock CS . The cash stock $CS(n+1)$ in year $n+1$ is given by

$$\begin{aligned} CS(n+1) &= (1 + \alpha) . CS(n) + E_s \\ &= (1 + \alpha) . CS(n) + e_s (1 - r - m) . S(n) \end{aligned}$$

Where α is the annual interest rate.

The mechanism of the economic growth is given in the following block diagram.



Some conclusions drawn from the models

- [1] The income of the nation $W+P$ and the consumer tax income T can be increased even if the total sales S , or GNP (gross national product) remains constant by reducing the cost R of the original raw materials and the cost M of production means.
- [2] The Income of the society $W+P$ and the consumer tax income give the source of the whole social expense X which can grow the social economy with clever investment for the future.
- [3] The critical condition for the growth is the investment ratio & time constant product $a \cdot tc$ be greater than 1. It means sufficient portion of the total expense should be invested for the future and the time constant of the effort result should be sufficiently large. That is, a true innovation must be achieved that creates new products for the society with long term effects.
- [4] The consumer tax is called an indirect tax but it is rather a tax directly collected through the exchange processes in the economy. It is a direct shift of a portion of the national income into the government sector. The expense of the consumer tax must be made effectively for the people.

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