

複素モデルによる非線形歪の補償と動作

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あらまし

増幅器の非線形歪は通信回路の深刻な特性劣化をもたらす事がある。振幅及び位相変化の激しい OFDM 変調などには特に劣化が甚だしい。非線形回路の線形等化法として Pre-distortion 方式が有効である[1,2]。Pre-distortion を行うには予め通信回線の非線形特性を知る必要があるがそれは多くの場合利用できるとは限らない。更に開ループ構成なので従来の pre-distortion 方式では通信路特性の経時変化に対応する事は困難である。そこで筆者は帰還路を設けて通信路の非線形歪を負帰還制御する方法を考案した[3]。この方法は対象となる通信路の非線形特性に関する詳細な予備知識が無くても広汎な通信系の非線形特性に適用できると共に経時変化にも自動的に追従して補償を行う事ができる。更に帰還系であるにも関わらず帰還路の遅延の影響がなく任意の伝送速度の変調信号に対応可能である。例えば長大な衛星通信の伝搬遅延を含む系にも適用可能である。

他方上記帰還型 Linearizer の動作設計の為には通信路の非線形性の正しい理解が有効である。筆者は増幅器の非線形特性を信号振幅によって変わる利得および遅延特性として捉える複素増幅器モデルを考案し解析を行った[4]。そこでは時間相関による信号成分の定義に基づき、入力信号成分とそれとは無相関な歪成分の動作解析を行った。また回路の浮遊容量のモデルに基づき増幅器の非線形性が生じる原因についても考察した。本稿においては上記複素モデルに基づき、Pre-distortion の機構を解析し、帰還型線形化回路の構成と動作について簡潔な表現を得たので報告する。

キーワード 非線形歪、AM/AM, AM/PM, 複素モデル、信号相関、無相関、信号遅延、浮遊容量、弾性法則

A Linearization Method of Non-linear Communication Paths based on A Complex Model

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Abstract

Nonlinear distortions of amplifiers can bring serious degradations in communication. A practical linearization method to reduce the nonlinear distortion is pre-distorter [1,2]. The conventional pre-distorters require the knowledge in advance about the nonlinear performances of the amplifiers in communication links, which is not available in many applications. As they are of open loop circuits, they cannot follow the changes of the nonlinear properties of the amplifiers in time either.

In order to solve those problems the author proposed a negative feedback linearizer for non-linear amplifiers based on a complex model a few years ago [3]. The negative feedback linearizer can not only cope with wide varieties of nonlinear properties of the amplifiers without the advance knowledge of the communications paths but also can follow the changes of parameters in time. The method is also applicable to cases with large paths delays.

The proposed method is based on a complex model of nonlinear amplifiers. The model is analyzed based on a stray capacity theory modeled as the balance between electrical and mechanical forces in the stray capacitors.

Keywords Non-linearity, Distortion, AM/AM, AM/PM, Correlation, Uncorrelated, Stray capacity, Hooke's law

1. Normalized Amplifier

An amplifier gives output $y(t)$ (t;time) caused by input $x(t)$. A linear amplifier with amplitude gain a gives $y(t) = a \cdot x(t)$.

If the output is $y_o(t)$ for a given input $x_o(t)$, the linear amplifier is normalized to that of the unit gain $a=1$ by replacing x/x_o by x and y/y_o by y . In the following our analysis is made on the normalized amplifier.

2. Taylor Expansion of Nonlinear Amplifier

If the function $y(x)$ is differentiable then it can be expanded in Taylor series.

$$y = x + a_2 \cdot x^2 + a_3 \cdot x^3 + \dots$$

where the coefficients a_2, a_3, \dots are constants.

In communication the transmitted signals are generally of the following modulations;

$$x(t) = p(t) \cdot \cos(\omega_c t) - q(t) \cdot \sin(\omega_c t)$$

where $p(t), q(t)$ are modulating signals carrying the transmitted data. ω_c is the angular frequency of the carrier.

Here the real function $x(t)$ can be expressed as the real part of a complex function $X(t)$:

$$x = \text{Re}\{X(t)\} = 1/2 \cdot (X + X^*)$$

$$X = c(t) \cdot e^{j \cdot \omega_c t}$$

where $c(t) = p(t) + j \cdot q(t)$ ($j^2 = -1$)

and X^* is complex conjugate of X .

The third term of the Taylor series is now calculated;

$$\begin{aligned} x^3 &= 1/8 (X^3 + 3X^2 \cdot X^* + 3X \cdot X^{*2} + X^{*3}) \\ &= 3/4 |c|^2 \cdot X + 1/8 (X^3 + X^{*3}) \\ &= 3/4 |X|^2 \cdot X + 1/8 (X^3 + X^{*3}) \end{aligned}$$

The frequency of the second term above is $3\omega_c$, namely the third harmonics which are removed by channel filters.

It can be shown only odd number of terms in the Taylor series expansions include components falling into the communication channel. Furthermore the $(2m+1)$ th term is of the form $|X|^{2m} \cdot X$.

The above representation of real functions by complex functions is generally used in communication engineering. We

can likewise set a complex function $Y(t)$ for $y(t)$ as;

$$y = \text{Re}\{Y\}$$

Then the amplifiers in communication can be expressed as ;

$$Y = G(|X|) \cdot X$$

where $G(|X|)$ gives the nonlinear complex gain.

3. Complex Model of Nonlinear Amplifiers

The general phenomena of amplifiers are ;

(1) AM/AM

The amplitude of the output is limited for the increasingly large input signal amplitude.

(2) AM/PM

The phase of the output signal delays against the input for the increasingly large input signal.

Based on the above phenomena, the general amplifier expression will be given in the following format;

$$Y = G(|X|) \cdot X$$

$$G(|X|) = 1 / (1 + \alpha \cdot |X|^2) / (1 + j \beta \cdot |X|^2)$$

$$(j^2 = j \cdot j = -1)$$

Where α, β are constants.

In the ranges

$$\alpha \cdot |X|^2 \ll 1,$$

$$\beta \cdot |X|^2 \ll 1,$$

The transfer function is approximately

$$\begin{aligned} G(|X|) &= 1 / (1 + (\alpha + j \beta) \cdot |X|^2) \\ &= 1 / \sqrt{1 + 2 \alpha \cdot |X|^2 + (\alpha^2 + \beta^2) \cdot |X|^4} \\ &\quad \cdot e^{j \arctan(\beta \cdot |X|^2 / (1 + \alpha \cdot |X|^2))} \end{aligned}$$

In terms of nonlinearity;

AM/AM ;

$$|G(|X|)| = 1 / \sqrt{1 + 2 \alpha \cdot |X|^2 + (\alpha^2 + \beta^2) \cdot |X|^4}$$

AM/PM ;

$$\text{Arg}(G(|X|)) = \arctan(\beta \cdot |X|^2 / (1 + \alpha \cdot |X|^2))$$

4. Saleh Model

Saleh model is widely used in linearizers designs[1,2].

In the phasor representation

$$X = r_i \cdot e^{j \theta}$$

$$Y = r_o \cdot e^{j(\theta + \phi)}$$

The Saleh model gives

$$\text{AM/AM}; \quad r_o = \alpha_1 r_i / (1 + \alpha_2 r_i^2)$$

$$\text{AM/PM}; \quad \phi = \beta_1 r_i^2 / (1 + \beta_2 r_i^2)$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants.

Note the proposed complex model is similar to Saleh model.

5. Stray Capacity Model

We now think about the cause of nonlinearity.

The proposed model is based on the stray capacity in the amplifier circuits (input side). Let the stray capacity C in the amplifier with internal resistor r so the transfer characteristics are

$$T = 1/(1+j\omega c.r.C)$$

If the capacity C changes according to the magnitudes of the input then some nonlinearity will result.

The stray capacity is modeled as the capacitors with the plates that can change the gap length according to the electric charges on the plates balanced with the elasticity force based on Hooke's law.

The capacitance C of the stray capacity is given by;

$$C = \epsilon_0 \cdot S / (d-u)$$

Where ϵ_0 is the dielectric constant of vacuum, S the area of the plates, $d-u$ is the gap lengths between the plates of the stray capacitors, of which d is the gap at no input signal and u is the variation due to the inputs to the amplifier.

When an input signal is applied to the amplifier the stray capacitor collects electric charge q which pulls the plates inwards by Coulomb force. The electric force is balanced by the elastic force following Hooke's law. Let the elasticity constant be k then the forces balance as;

$$k \cdot u = q \cdot q / (d-u)^2 / (4\pi \cdot \epsilon_0)$$

The right hand side suggests the force is proportional to the square of the input amplitude $|x(t)|^2$. It is also probable $d \gg u$ hence u will be proportional to $|x|^2$.

Based on the above discussion we suppose the stray capacity is expressed as ;

$$C = C_o \cdot (1 + c \cdot |x|^2) \quad (c; \text{constant})$$

Then the transfer characteristics is given by

$$\begin{aligned} T &= 1/(1+j\omega c.r.C) = 1/(1+j\omega c.r.C_o \cdot (1 + c \cdot |x|^2)) \\ &= 1/(1+j.d \cdot (1 + c \cdot |x|^2)) \quad (d = \omega c.r.C_o) \\ &= 1/\sqrt{\{1+d^2 + 2d.c \cdot |x|^2 + d.c^2 \cdot |x|^4\}} \\ &\quad \cdot e^{-j \cdot \arctan\{d \cdot (1 + c \cdot |x|^2)\}} \end{aligned}$$

In the expression in terms of nonlinear elements;

$$\sqrt{1+d^2} \cdot T = 1/\sqrt{\{(1+f_1 \cdot |x|^2 + f_2 \cdot |x|^4)\}} \cdot e^{-j \cdot \arctan\{d \cdot (1 + c \cdot |x|^2)\}}$$

where,

$$f_1 = 2d.c/(1+d^2),$$

$$f_2 = d.c^2/(1+d^2)$$

The nonlinearity of the model;

AM/AM

$$|G(|X|)|^2 = 1/(1+f_1 \cdot |x|^2 + f_2 \cdot |x|^4)$$

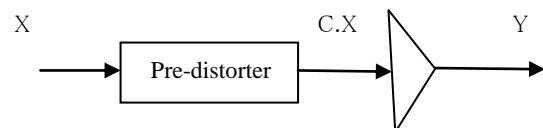
AM/PN

$$\text{Arg}\{G(|X|)\} = -\arctan\{d \cdot (1 + c \cdot |x|^2)\}$$

The behavior of the proposed model is similar to those described in the preceding sections.

6. Open Loop Pre-distorter

The structure of an open loop pre-distorter is simple; Amplifier



We now describe the mechanism of the linearization.

Without linearization;

The output Y' of the amplifier is given by multiplication of input X with the nonlinear gain $G(|x|)$;

$$Y' = G(|x|) \cdot X$$

With linearization;

The input signal to the amplifier is compensated by complex multiplication with the control signal $C(|x|)$. Then the output of the amplifier gets;

$$\begin{aligned} Y &= Y'(C(|X|) \cdot X) \\ &= G(|C(|X|) \cdot X|) \cdot C(|X|) \cdot X \end{aligned}$$

The linearization is achieved if

$$G(|C(|X|) \cdot X|) \cdot C(|X|) = 1$$

Example 1 ;

For $|G(|X|)| = 1/\sqrt{1 + \alpha \cdot |X|}$

Then $C(|X|) = 1/\sqrt{1 - \alpha \cdot |X|^2}$

This is the case the amplifier has no saturation.

Note the range of linearization is limited to;

$$|X| < 1/\sqrt{\alpha}$$

Example 2;

For $G(|X|) = 1 / \{1 + \alpha \cdot |X|^2\}$

Then $C(|X|) = \{1 - \sqrt{1 - 4\alpha \cdot |X|^2}\} / \{2\alpha \cdot |X|^2\}$

Note the dynamic range is $|X| < 1/2\sqrt{\alpha}$

This is the case output Y' is saturated at

$$|X| = 1/\sqrt{\alpha},$$

$$|Y'| = \sqrt{\alpha}/2$$

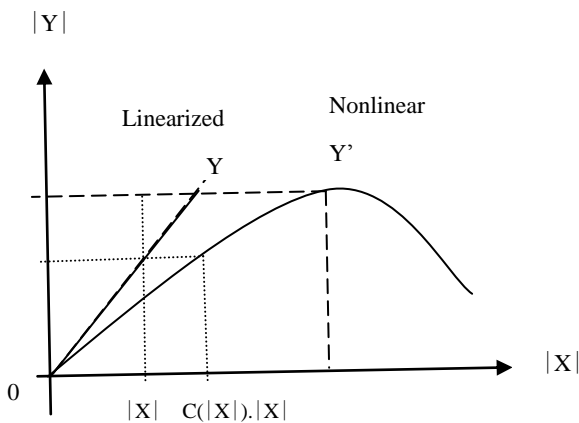
The linearizer achieves the maximum power out of the amplifier

at $|X| = 1/2/\sqrt{\alpha},$

$$|Y| = \sqrt{\alpha}/2$$

where $C = 2.$

The mechanism of linearization is depicted in the following figure.



Example3;

For $G(|X|) = 1 / \{1 + j\beta |X|\}$

Then $C(|X|) = Co / (\beta |X|)$

where Co is an arbitrary constant for compensation of AM/PM/.

Then the linearized output is

$$Y = X/\sqrt{1 + Co^2}$$

The above example suggests that in order for a complete linearization, the amplitude and phase must be independently controlled.

In practice the exact characteristics of the nonlinearity are not known in advance. Therefore we need to assume some simple model such as Saleh and determine the parameters by a few tests.

The accuracy of linearization is limited by the differences between the true characteristics of the amplifier and the adopted model. The changes of parameters in time cannot be followed by the open loop pre-distorter either.

7. Closed Loop Pre-distortion

We review the linearizer equation

$$Y = Y'(C(|X|).X)$$

$$= G(|C(|X|).X|).C(|X|).X$$

It can be viewed as a two stages amplifiers composed of

First stage; $Y1 = C(|X|).X$

Second stage $Y = G(|Y1|).Y1$

The total function of the amplifiers is

$$Y = G(|Y1|).C(|X|).X$$

Our objective is to achieve

$$\begin{aligned} Y/X &= G(|Y1|).C(|X|) \\ &= 1 \end{aligned}$$

If we take logarithm of the terms, our goal is

$$\begin{aligned} [Y] - [X] &= [G] + [C] \\ &= 0 \end{aligned}$$

where $[Y]$ is logarithm of $Y.$

$$[Y] = [|Y|] + j \cdot \langle Y \rangle$$

where

$$\langle Y \rangle = \arg(Y), \text{ or}$$

$$Y = |Y| \cdot e^{j \cdot \langle Y \rangle}$$

Then our goal is achieved if

$$[|Y|] - [|X|] = [G] + [C] = 0$$

$$\langle Y \rangle - \langle X \rangle = \langle G \rangle + \langle C \rangle = 0$$

The left hand side $[Y]-[X]$ is the error that is to be minimized to zero by control of $[C].$

In order to achieve our goal we need

- Error detector that gives $[Y]-[X] = [Y/X]$
- Pre-distorter that achieves $C.X$ for input X
- Amplifier $G(|X|)$ to be linearized.
- Control data memory that stores $[C(|X|)]$.
 Note the memory is addressed by $|X|$.
- Control Data Generator that gives $[C]$

The error integral method can achieve the objective ;

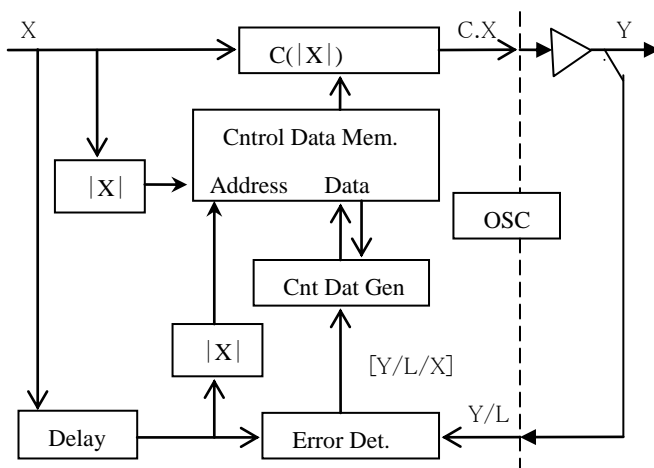
$$[C](n+1) = [C](n) - k.[Y/X](n)$$

where $[C](n)$ is the control data stored in the memory in the n -th control. The data is renewed by replacing the content of the memory by $[C](n+1)$.

k is the loop gain of the control loop for generating $[C(|X|)]$. Generally k must be < 1 for stability of the loop.

- Feedback loop from output of the amplifier to the control circuit to generate the error signal $[Y/X]$.
- Delay circuit for compensating the delay of the feedback loop to the error detector.

A block diagram of the feedback linearizer is depicted in the following figure.



A few notes will be helpful for understanding the operation of the proposed method.

The portion of the figure to the right of the dotted line operates in RF bands. Those to the left side operate in

baseband in digital complex format. The lines \rightarrow are complex signals and \longrightarrow are real data pointing to the address of the Control Data Memory.

The signals shown by \longrightarrow are real signals in the RF bands. The dotted line depicts the frequency converters; Quadrature Amplitude Modulator (QAM) for the up-converter and Quadrature Amplitude Demodulator (QAD) for the down converter. OSC stands for the oscillators for generating complex carrier signals $e^{j(\omega c.t)} = (\cos(\omega c.t), \sin(\omega c.t))$ and other timing signals.

The feedback loop branches a small portion of the amplifier output by a directional coupler for the linearizer operation. Let us depict the loss of the feedback loop by L then the linearizer functions to achieve

$$[Y/L/X] = [Y] - [L] - [X] = 0$$

$$[Y/X] = [L] \quad ; \text{ Total gain of the linearized transmitter}$$

The reference phase is achieved for small input X well in the linear operation range. The reference phase needs to be removed for phase error detection to get the phase error due to the nonlinear effects in the power amplifier.

Control Data Memory needs to be initially set with some appropriate data. It may be also necessary to interpolate the control data over some range of addresses based on each measurement cycle. For those purposes proper complex models of the nonlinear amplifiers will be useful.

Because it contains the delay compensation the proposed method is applicable to systems with large transmission delays, such as satellite communications.

Conclusion

A complex model of a nonlinear power amplifier is proposed. The causes of nonlinear effects are studied by a stray capacity model which gives similar behaviors as Saleh and other models herein proposed.

Some examples of inverse functions for the open loop pre-distorter are shown. The operational limits of open loop pre-distorters are discussed and a closed loop pre-distorter is proposed with the theory of its operations.

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Symbols

Some unfamiliar mathematical symbols are explained here.

- (1) $X \cdot Y = X \cdot Y$ is the product of X and Y.
- (2) X^n is X to n as in the n-th term in Taylor expansion.
- (3) For a complex variable X

$$X = |X| \cdot e^{j\langle X \rangle}$$

where

$|X|$ is the absolute value of X.

$\langle X \rangle$ is the argument or phase of X.

j is imaginary number unit; $j^2 = -1$

- (4) $[X]$ is logarithm of X

$$[X] = [|X|] + j\langle X \rangle$$